Progress and Goals for simulations of HIT

K.D. Morgan and the HIT Team

PSI-Center Annual Meeting, July 26 2018
Outline

• Description of the HIT-SI/HIT-SI3 devices and how we implement them in NIMROD

• Results of a scan of injector frequency on HIT-SI

• Results of a scan of injector temporal phasing on HIT-SI3

• Results of high-gain simulations of larger devices

• Conclusions and future plans
The HIT-SI and HIT-SI3 devices are bowtie-shaped spheromaks.

- A central bowtie-shaped flux conserver is driven by three inductively driven semi-toroidal injectors.
Steady Inductive Helicity Injectors (SIHI) sustain the spheromak through purely inductively driven helicity injection

- Each injector is a semi-toroid driven like an oscillating RFP, with the ‘toroidal-field’ coils (white) driving flux and the ‘central-solenoid’ coils (colored) driving current.

- Injectors are driven out of phase, providing constant helicity injection.
Temporal phasing of the injectors provides control over the perturbation spectrum.

• The helicity injected by a single injector is given by ($\omega = 2\pi f_{inj}$):

$$\dot{K}_{inj} = 2V_{inj}\psi_{inj} = 2V_0\psi_0 \sin^2(\omega t)$$

• HIT-SI operated two injectors 90° out of phase for constant injection:

$$\dot{K}_{inj} = 2V_0\psi_0(\sin^2(\omega t) + \cos^2(\omega t)) = 2V_0\psi_0$$

• HIT-SI3 operated out of phase ($\delta\phi = 60,120$):

$$\dot{K}_{inj} = 2V_0\psi_0(\sin^2(\omega t) + \sin^2(\omega t + \delta\phi) + \sin^2(\omega t + 2\delta\phi)) = 3V_0\psi_0$$

• HIT-SI3 operated in phase ($\delta\phi = 0$): $\dot{K}_{inj} = 6V_0\psi_0 \sin^2(\omega t)$
HIT-SI and HIT-SI3 discharges begin with the formation of a non-axisymmetric injector state.

- Shown is a Taylor state of the injectors in HIT-SI3:
  \[ \nabla \times B = \lambda B \]

- \[ \frac{I_{inj}}{I_{tor}} = 0 \]

- These states are initially driven before a relaxation event, where the spheromak forms.

Figure courtesy Chris Hansen
HIT-SI and HIT-SI3 discharges relax into an axisymmetric state, with the non-axisymmetric injectors still operating.

• Shown is a composite Taylor state of HIT-SI3:
  \[ \nabla \times B = \lambda B \]

• \( \frac{I_{inj}}{I_{tor}} = 6 \)

• When the axisymmetric object has formed, the injector fields tend to wrap around near the edge of the device, while the spheromak is near the core.

Figure courtesy Chris Hansen
All operating schemes of injectors follow similar discharge evolution.

Relaxation event: formation of axisymmetric \( (n=0) \) spheromak.

Initial build-up of non-axisymmetric injector state.

Sustainment of axisymmetric \( (n=0) \) spheromak.
Temperature evolution shows transition from injector state to spheromak state (HIT-SI)

- Red 3-D contour is $T \sim 0.9T_{\text{max}}$
- Early on in the simulation, the injector magnetic fields dominate the temperature profile.
- At higher current gain, the axisymmetric fields begin to dominate the temperature profile, with warm injector filaments near the edge.

\[
\frac{I_{\text{tor}}}{I_{\text{inj}}} = 1
\]
\[
\frac{I_{\text{tor}}}{I_{\text{inj}}} = 3
\]
Temperature evolution shows transition from injector state to spheromak state (HIT-SI3 - 120)

- Red 3-D contour is $T \sim 0.9T_{\text{max}}$
- Early on in the simulation, the injector magnetic fields dominate the temperature profile.
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Temperature evolution shows transition from injector state to spheromak state (HIT-SI3 - 60)

- Red 3-D contour is $T \sim 0.9T_{\text{max}}$
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Temperature evolution shows transition from injector state to spheromak state (HIT-SI3 - 0)

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• Early on in the simulation, the injector magnetic fields dominate the temperature profile.
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$\frac{I_{\text{tor}}}{I_{\text{inj}}} = 1$

$\frac{I_{\text{tor}}}{I_{\text{inj}}} = 3$
During sustainment spheromak moves at injector frequency \( f_{inj} \)

- Shown are contours of \( RB_\phi \) at a current gain of \( \frac{I_{tor}}{I_{inj}} = 3.5 \).
- The spheromak experiences periodic motion related to injectors pushing it around.
NIMROD is used to solve the equations of Hall-MHD

• Several models are examined, with the zero-\(\beta\) model assuming \(\frac{\partial n}{\partial t} = 0\), \(\frac{\partial T}{\partial t} = 0\), and \(\nabla p = 0\). The finite-\(\beta\) model additionally involves the terms in red.

\[
\frac{\partial n}{\partial t} = -\nabla \cdot n\mathbf{V} + D\nabla^2 n
\]

\[
\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = \frac{1}{\rho} \left( J \times \mathbf{B} - \nabla p - \nabla \cdot \mathbf{\Pi} \right)
\]

\[
\frac{n}{\gamma - 1} \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) T = -p \nabla \cdot \mathbf{V} - \mathbf{\Pi} : \nabla \mathbf{V} - \nabla \cdot \mathbf{q} + \eta J^2
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]

\[
\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J} + \frac{J \times \mathbf{B} - \nabla p_e}{ne} + \frac{m_e}{ne^2} \frac{\partial J}{\partial t}
\]
The injectors are modelled as boundary conditions on the domain

- The ‘Bow-Tie’ flux conserver is modeled, neglecting fine experimental features.
- The injector fields on the boundary of the flux-conserver are imposed, red and blue regions show the locations of flux/current entering and leaving the volume.
- A thin (1 mm) layer of high resistivity ($\eta_{\text{edge}} \sim 10^5 \eta_{\text{plasma}}$) is used to simulate the insulating layer on the experiment, further imposing a pseudo-boundary condition of $J_\perp = 0$
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HIT-SI’s final run campaign focused on scan of $f_{\text{inj}}$

• HIT-SI was operated at $f_{\text{inj}} = 14.5, 36.8, 53.5, \text{ and } 68.5 \text{ kHz.}$

• At ‘high-frequency’ several changes in performance were seen:
  • Magnetic axis shifted outward, equilibrium fits give $\beta_{\text{vol}} \sim 25\%$
  • Increased toroidal symmetry was seen on surface magnetic probes
  • Higher current gain
  • Injector impedance increases linearly with $f_{\text{inj}}$
The toroidal mode structure of the perturbation depends on $f_{inj}$

![Relaxation Dynamics, $f_{inj} = 14.5$ kHz](image1)

0 – $\beta$

![Relaxation Dynamics, $f_{inj} = 68.5$ kHz](image2)

Finite-$\beta$
Experiment agrees with increased $n = 2$ activity during startup

• Shown are the toroidal mode structures from surface probes located on the outboard midplane.

• The ratio of $n = 2$ to $n = 1$ energy is not as severe, but as we will see in the simulation the mode is primarily located away from the wall.

Hossack, et al, 2017
\[ n = 2 \] injector state consists of four helical tubes aligned with the injector mouths

- The injector state that forms prior to relaxation in the high-\( f_{\text{inj}} \) case is primarily \( n = 2 \) shaped with a small \( n = 1 \) perturbation.
- Shown on the right is the magnetic fields in the midplane during this period.
$n = 2$ injector state consists of four helical tubes aligned with the injector mouths
n=2 spheromak formation occurs through merging of two of these flux tubes (two different simulations).
n=1 formation is less visible on midplane, but can see difference (two different simulations).
Transition frequency is near Alfven transit time of injector current path.

• Using typical formation values of $B = 10 \text{ mT}$ and $n = 1e19 \text{ m}^{-3}$, we obtain $\frac{1}{\tau_A} \sim 30 \text{ kHz}$ for path between injector mouths, which is near where the experiment sees this transition.

• Should be able to confirm this using NIMROD, zero-$\beta$ simulations are relatively quick to run through formation and capture the effect.
  • Initial simulations show weird behavior near this ‘transition’ frequency, so more unclear yet.
To study the sustainment phase we begin with an equilibrium that decays.

- A Taylor state at gain of 4 was used as the initial condition.
- Injectors driven at 8 different frequencies (5,15,30,45,60,68,80,100) kHz at constant settings were used.
- Parameters were taken from high frequency ($f_{inj} = 68.5$ kHz) experimental discharges.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_0$ (m$^{-3}$)</td>
<td>$7.5 \times 10^{18}$</td>
</tr>
<tr>
<td>$T_{wall}$ (eV)</td>
<td>2</td>
</tr>
<tr>
<td>$I_{inj}$ (kA)</td>
<td>8.5</td>
</tr>
<tr>
<td>$\psi_{inj}$ (mWb)</td>
<td>0.5</td>
</tr>
<tr>
<td>$\nu$ (m$^2$/s)</td>
<td>550</td>
</tr>
<tr>
<td>$D$ (m$^2$/s)</td>
<td>250</td>
</tr>
<tr>
<td>$V_{inj}$ (km/s)</td>
<td>20</td>
</tr>
<tr>
<td>$\eta / \mu_0$ (m$^2$/s)</td>
<td>$423 T^{-3/2}$</td>
</tr>
</tbody>
</table>
Current gain raises with $f_{\text{inj}}$

• Current gain is primarily determined by $\eta$ in the zero-$\beta$ model.

• With pressure effects, the gain does not increase as fast as temperature does.
NIMROD captures $\beta$ increase seen at higher frequencies.

- There appears to be a threshold frequency where the beta increase starts occurring.
- The beta limit of the equilibrium increases linearly with injector frequency, is not sensitive to parallel thermal conduction.
Density and temperature profiles change with frequency

• Shown are profiles of $n$, $T$, and $p$ on the midplane time averaged over several injector cycles.

• The plasma pressure is flat through most of the volume, but elevates above the wall value.
Current centroid change is not captured in single-temperature model.

- The mean center of the edge poloidal magnetic-field is calculated:
  \[
  < R > = \frac{\sum_n B_{pol} R_n}{\sum_n B_{pol}}
  \]

- \(<R>\) is calculated at four toroidal locations, shown is average of those measurements, error bars represent range of measurement.

- While an outward shift is seen in the finite-beta simulations, it is not shifted outward as much as the experiment.
The kinetic energy in the volume increases linearly with $f_{inj}$

- The kinetic energy of the simulation primarily manifests in the $n = 1$ symmetry of the injector drive.
- Injector impedance scales linearly with $f_{inj}$, for constant $I_{inj}$ the higher frequency shots have larger injector power.
The kinetic energy in the volume increases linearly with $f_{\text{inj}}$

- The kinetic energy of the simulation primarily manifests in the $n = 1$ symmetry of the injector drive.
- The magnitude of the kinetic energy increases linearly with $f_{\text{inj}}$.
- Viscous heating with these flows becomes larger than Ohmic heating at high-$f_{\text{inj}}$. 
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HIT-SI3 uses 3 injectors to provide variation in toroidal mode structure

- The three injectors are driven temporally out of phase with each other.
- Depending on the relative phasing, vastly different perturbation toroidal spectra are driven.
- For comparison, HIT-SI drove a perturbation that was primarily n=1.
Applied perturbations have toroidal spectrum based on injector phasing.

- The applied perturbation has two extrema:
  - Purely harmonics of n=3. (0-0-0)
  - Combination of n=1,2,4,5,… (0-120-240)
- In full simulation the energies mostly collect in n=1,2, and 3.

- Characterize the parameter space as the amount of perturbation applied in the n=3 harmonics.
- Experiment and simulation have operated at several locations, future operations intend to scan space in more detail.
9 simulations were performed to look at 3 injector phasings

- At each of the 3 experimental phasings (0-0-0, 0-120-60, and 0-120-240), 3 simulations were performed at $f_{inj} = 14.5$ kHz, matching $I_{inj}$ and $\psi_{inj}$ waveforms.
- Two zero-$\beta$ simulations varied the viscosity ($\nu_\parallel = \nu_\perp$ and $\nu_\parallel = 11\nu_\perp$)
- One finite-$\beta$ simulation using $\nu_\parallel = 11\nu_\perp$ and Braginskii thermal conduction coefficients

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulation</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_e$ (kg)</td>
<td>$9.1 \times 10^{-29}$</td>
<td>$9.1 \times 10^{-31}$</td>
</tr>
<tr>
<td>$n_e$ (m$^{-3}$)</td>
<td>$2 \times 10^{19}$</td>
<td>$1 - 10 \times 10^{19}$</td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
<td>12</td>
<td>13 $\pm$ 7</td>
</tr>
<tr>
<td>$\eta/\mu_0$ (Spitzer) (m$^2$/s)</td>
<td>8.8</td>
<td>25-9</td>
</tr>
<tr>
<td>Viscosity ($\nu_\parallel$) (m$^2$/s)</td>
<td>3300</td>
<td>300-5000</td>
</tr>
<tr>
<td>Viscosity ($\nu_\perp$) (m$^2$/s)</td>
<td>300</td>
<td>100-500</td>
</tr>
</tbody>
</table>
The injector current and flux waveforms are taken from the experiment

- The experimental input to the spheromak sustainment is done through driving an injector flux and current.
- The NIMROD simulations use a piecewise linear function to approximate this input waveform:

\[ f(t) = A(t) \cos(\omega_{\text{inj}} t + \delta \phi(t)) \]

- Comparisons of injector waveforms are seen on the right.
Bulk parameters show reasonably good agreement:

- The toroidal plasma current serves as a good measure of formation and evolution of the spheromak.
- The simulations tend to underestimate the growth early in time and overestimate later in time.
- Depending on the injector phasing, parallel viscosity can both increase and decrease the toroidal current by $\sim 10\%$. 

![Graphs showing toroidal currents over time](image-url)
The toroidal modal structure of the injector perturbation agrees reasonably well

- The toroidal mode structure is calculated experimentally from surface probes on the outboard midplane.
- Shaded region is average of 5 similar discharges.
- The spectrum from probes shows similar behavior, with disagreements from experiment likely being fine-scale features of the experiment that are not captured
  - Holes in flux conserver for diagnostic access
  - Finite resistivity of wall
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\[ \text{Toroidal Modes, 0-120-60} \]

\[ \langle B_{tor} \rangle (T) \]

\[ 0.002 \rightarrow 0.012 \]

\[ 0.006 \rightarrow 0.008 \]

\[ n=1 \]
\[ n=2 \]
\[ n=3 \]

Isotropic
Anisotropic
Finite-\( \beta \)
The toroidal modal structure of the injector perturbation agrees reasonably well

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- Shaded region is average of 5 similar discharges.
- The spectrum from probes shows similar behavior, with disagreements from experiment likely being fine-scale features of the experiment that are not captured
  - Holes in flux conserver for diagnostic access
  - Finite resistivity of wall
Internal magnetic oscillations show agreement (0-120-240)
Internal magnetic oscillations show agreement (0-120-60)
Internal magnetic oscillations show agreement (0-0-0)
NIMROD provides insight to the volumetric energy spectrum

- Using confidence gained from favorable comparisons with the surface and internal probe measurements of the perturbation, we can examine the volume integrated spectrum.
- We see the volume integrated spectrum is different from the surface probe measurement, to give an idea of the poloidal structure.
- At 0-120-240, the $n = 1$ is dominant at the edge and $n = 2$ is dominant in the core.
NIMROD provides insight to the volumetric energy spectrum

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• We see the volume integrated spectrum is different from the surface probe measurement, to give an idea of the poloidal structure.

• At 0-120-60, the $n = 2$ is dominant at the edge and $n = 1$ is dominant in the core.
NIMROD provides insight to the volumetric energy spectrum

• Using confidence gained from favorable comparisons with the surface and internal probe measurements of the perturbation, we can examine the volume integrated spectrum.

• We see the volume integrated spectrum is different from the surface probe measurement, to give an idea of the poloidal structure.

• At 0-0-0, the perturbation is harmonics of $n = 3$, which shows up clearly on bot
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Large bow-tie flux conserver shows closed-flux formation at gains above 6

• Running a full non-linear simulation with HIT-SI injectors and a larger geometry ($R_0 = 2.5R_{hitsi}$) and lower resistivity ($T_e \sim 70$ eV) sees the formation of closed flux-regions

• These closed flux periods persist for 10-20 injector periods before opening up.
The motion of the equilibrium is clear when flux surfaces are closed.
Poloidal flux inside closed flux region decays resistively before being opened due to kink-instability

• The poloidal flux inside the separatrix decays at $\tau_{L/R}$, causing the $q$ profile to become more peaked.

• Eventually, after $|q|_{max} > 1$, the flux surfaces open and a refluxing event occurs before the flux surfaces close back up.

• These cycles are periodic, with each closed-flux period lasting $\sim 10 - 20$ injector cycles.
Midplane puncture plots show this process.
Can restart simulation with finite-$\beta$ model to see temperature evolution.

- Initialized with closed-flux equilibrium and uniform temperature and density profiles.
- Have run this with both single-$T$ and split-$T$ models.
- Total energy of system is decreasing, have not run far enough to reach steady state behavior.
- Electrons become hotter than ions during closed flux period.
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A variety of steps can be taken next on HIT-SI modeling

• Two places exist to improve the pressure modeling:
  • The Meier-Shumlak neutral fluid model can provide an avenue for more-accurate physics near the edge of the plasma, where higher neutral densities are expected.
  • Split ion-electron temperature models are likely important. Experimental results and initial simulations expect $\frac{T_i}{T_e} \sim 3-4$ at high-injector frequency.

• Variation of experimental geometry
  • HIT-SI3 showed an avenue to explore different perturbation spectra, can use NIMROD to test different configurations.
  • Equilibrium modeling has suggested flux-conserver shapes with higher $\beta$ equilibria than the bow-tie shape. Some of these work well with Akcay’s Boolean sum interpolation mesh generator, some have issues.
Boundary condition routines have been expanded, allowing variety of injector designs

• Now have streamlined process for implementing injector geometry.

• Can use arbitrary combination of injector mouths located on top, bottom, and midplane of geometry.

• James Penna is further exploring role of the imposed perturbation spectrum on evolution.
Conclusions

• NIMROD has been used to explore the formation and sustainment phases of HIT-SI/HIT-SI3 discharges.

• Finite-\(\beta\) effects are important in obtaining agreement with experimental results.
  • Early indications say split-temperature models are also important.

• Computations become ‘easier’ to perform at higher current amplification, due to increased amplitude of toroidally symmetric fields.

• More generalized boundary condition forms have been developed to test additional injector configurations.
Backup Slides
The HIT-SI device was a bowtie-shaped spheromak

- A central bowtie-shaped flux converter driven by two inductively driven semi-toroidal injectors.
Conclusions and next steps

• Extended MHD models can capture the dominant physics of high-frequency HIT-SI discharges, though the most accurate models are computationally demanding.
  • Pressure gradients play an important role in both the formation and sustainment of HIT-SI plasmas.
  • Experimental observations of increased $\beta$ with increased $f_{inj}$ have been observed in simulations.

• Improvements to the model are required to improve agreement with experimental results:
  • Anistropic thermal conduction becomes exceptionally stiff, thought to be related to high-$\eta$ edge layer adding additional noise on $B$.
  • Injector pressure is incorrectly modeled due to boundary condition nature, comparisons with full-3D codes that additionally model injectors should lead to improvements. See poster JP11.00141 tomorrow morning by Thomas Benedett.
Steady Inductive Helicity Injectors (SIHI) sustain the spheromak through purely inductively driven helicity injection

• Each injector is a semi-toroid driven like an oscillating RFP, with the ‘toroidal-field’ coils (blue) driving flux and the ‘central-solenoid’ coils (red) driving current.

• Injectors are driven out of phase, providing constant helicity injection.
HIT-SI3 studies the formation and sustainment of the spheromak state

$I_{\text{tor}} = 0$

$I_{\text{tor}} = 6I_{\text{Inj}}$
The injectors are modelled as boundary conditions on the domain

- The ‘Bow-Tie’ flux conserver is modeled, neglecting fine experimental features.
- The injector fields on the boundary of the flux-conserver are imposed, red and blue regions show the locations of flux/current entering and leaving the volume.
- A thin (1 mm) layer of high resistivity \( \eta_{edge} \sim 10^5 \eta_{plasma} \) is used to simulation the insulating layer on the experiment, further imposing a pseudo-boundary condition of \( J_\perp = 0 \)
Boundary condition on velocity helps reduce required particle diffusivity

• Previous simulations required a large particle diffusivity $(D > 1000\text{m}^2/\text{s})$ which damped density dynamics significantly.

• Experimental measurements indicate that the $V_\perp = 0$ condition used previously is incorrect.

• Setting $V_\perp \propto B_\perp$ with peak value of $V_0 = 20 \text{ km/s}$ allows us to reduce $D$ by a factor of 4.
We can calculate $P_{inj}$, $V_{inj}$, and $Z_{inj}$ from energy balance.

• Using energy balance to isolate the injector power:
  \[
  \frac{\partial E_B}{\partial t} = P_{inj} - \int E \cdot J dV
  \]

• Boundary condition magnitudes are set to provide desired $I_{inj}$ and $\psi_{inj}$, can find effective voltage and impedance from:
  \[
  V_{inj} = \frac{P_{inj}}{I_{inj}}
  \]
  \[
  Z_{inj} = \frac{P_{inj}}{I_{inj}^2}
  \]

• This quantity is volume averaged, closest analog experimentally is the quadrature sum values:
  \[
  V_{inj} = \sqrt{V_x^2 + V_y^2}
  \]
NIMROD calculated $Z_{inj}$ sees linear dependence on both $j/n$ and $\omega_{inj}$ predicted by IDCD

- IDCD model (Jarboe, 2014) gives the following equation:
  $$Z_{inj} = \frac{C_1 \mu_0 R_0}{\pi} \left( \frac{\lambda_{inj} I}{8\lambda a^3 ne} + \frac{C_2 2\omega}{a\lambda} \right)$$
- NIMROD sees both linear dependences, though disagrees with fitting of $C_1$ and $C_2$.
- Disagreement is likely combination of density disagreement (particle diffusion) and the fact that injectors are not completely modelled.

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<th>NIMROD</th>
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<td>$C_1$</td>
<td>0.8</td>
<td>0.08</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1.1</td>
<td>2.7</td>
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- NIMROD sees decreasing values up to a limit, then it starts increasing.
- Additionally, absolute value of fluctuations is smaller in NIMROD
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• Additionally, absolute value of fluctuations is smaller in NIMROD
Increased injector impedance may explain increase in gain and $\beta$

- In these simulations $I_{inj}$ and $\psi_{inj}$ were held constant, but since $Z_{inj} \propto \omega_{inj}$ the injector power similarly scales $P_{inj} \propto \omega_{inj}$
- Have started working on a 0-D power balance model related to this.
The internal magnetic profile differs, experiment sees outward shift of magnetic axis

- The internal magnetic profile is obtained by time-averaging the internal magnetic probe at the injector frequency.
- NIMROD tends to disagree with the location of the magnetic axis by ~1 cm, leading to a difference in the value of $\beta$ derived from equilibrium fitting.
- The Grad-Shafranov value of $\beta$ agrees with the actual value from the simulation within 10%.
Current centroid confirms observation of increased $\beta$

- The mean center of the edge poloidal magnetic-field is calculated:
  \[
  \langle R \rangle = \frac{\sum_n B_{pol} R_n}{\sum_n B_{pol}}
  \]
- $\langle R \rangle$ is calculated at four toroidal locations, shown is average of those measurements.
- The zero-$\beta$ simulations see $\sim 1$ cm offset in location of this centroid, while the finite-$\beta$ simulations capture it. This shift is thought to be caused by $\langle \beta \rangle_{vol} \sim 10 - 15\%$
Current centroid confirms this observation

- The mean center of the edge poloidal magnetic-field is calculated:

\[ < R > = \frac{\sum_n B_{pol} R_n}{\sum_n B_{pol}} \]

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Additional comparisons are done on fluid parameters

• A set of optical diagnostics allow for the measurement of \( n, V, \) and \( T \).

• Two Ion Doppler Spectroscopy arrays allow measurement of ion flow and temperature

• A single interferometer chord allows measurement of density oscillations.
The Ion Doppler Spectrometer (IDS) system provides comparisons of flow profile

• The IDS system calculates the toroidal flow profile by taking the difference between two chords of identical impact parameter, pointing opposite direction toroidally.

• The flow profile obtained by NIMROD appears to more closely match the O II flow profile, more so than the C III flow profile.

• These velocities are much smaller than toroidal rotation at the injector frequency would be.
The Ion Doppler Spectrometer (IDS) system provides comparisons of ion temperature profile

• IDS is able to provide a line integrated temperature for two different ion species.

• Experimentally the O II temperature is seen to be lower than the C III temperature, with the NIMROD temperature closer to the O II profile.

• Additionally, the actual midplane T profile is smoothed significantly by the line-averaging of IDS.
Temperature time evolution provides helpful insights for future simulations

• The experiment sees a relatively flat time evolution, starting hotter than the initial condition used in the simulation.

• The simulation assumes a cold initial temperature then heats up throughout the shot.

• The undervaluing of $I_{tor}$ early in time and overvaluing late in time are likely results of differences in the evolution of $T$. 
The Far-Infrared (FIR) interferometer provides comparisons of density oscillations

- Density oscillations of order \( \frac{\delta n}{n} \sim 50\% \) are seen experimentally, oscillating at the injector frequency.
- The large particle diffusivity \( (D = 1000 \text{ m}^2/\text{s}) \) appears to damp density oscillations that are seen experimentally, though the frequency of oscillations are still seen.
Outward shift of magnetic axis is seen experimentally at high-$f_{inj}$

- Equilibrium fits of experimental discharges at low and high $f_{inj}$ are shown.
- The high-$f_{inj}$ discharges seen an outwardly shifted magnetic axis, which equilibrium fits resolve with higher $\nabla p$.  

![Graph showing the relationship between B/I and major radius, as well as B and poloidal angle.](image-url)
The current centroid shows both an outward shift and increased symmetry at higher $f_{inj}$

- The mean center of the edge poloidal magnetic-field is calculated:
  \[
  \langle R \rangle = \frac{\Sigma_n B_{pol} R_n}{\Sigma_n B_{pol}}
  \]

- The filled region indicates range of 4 toroidal measurement locations, with the line indicating the average.

- Higher $f_{inj}$ has experimentally lead to an outward shift and increased toroidal symmetry.
Single temperature simulations show increased symmetry, but not outward shift

- Higher $f_{\text{inj}}$ formation simulations see increased symmetry compared with lower $f_{\text{inj}}$.
- The outward shift is not seen.
Pressure gradients are required to capture symmetrization

- Shown is two calculations at high frequency. Finite-$\beta$ and Zero-$\beta$.
- Initial expectation is that increased current gain ($\frac{I_{\text{tor}}}{I_{\text{inj}}} \sim \frac{B}{\delta B}$) should correlate with increased toroidal symmetry.
- Instead, pressure gradients increase toroidal symmetry.
- Gradients in $\eta$ ($\sim T_e^{\frac{3}{2}}$) thought to be responsible.
\( T_e = T_i \) assumption can be relaxed.

- Experimental measurements (IDS, Langmuir probe) indicate that \( T_i \sim 2T_e \).
- Extend MHD model to include separate temperatures for each species.
- NIMROD results agree that \( T_i \sim 2T_e \).
Results are promising, but computationally intensive

• Relaxation occurs faster, outward shift of the magnetic axis appears, and $\beta$ is increased.

• Unfortunately, post relaxation the computation becomes stiff.
Simulations assuming uniform $T_e$ are able to capture outward shift.

- $T_e$ profile is observed to be flat over most of volume, so simulations are conducted evolving only $T_i$.
- Shown is single-temperature simulation from before and uniform $T_e$ simulation.
- $T_e = 13$ eV is assumed, giving same $\eta$ as zero-$\beta$ case. Current gain is lower, indicating that $\nabla p$ reduces current gain.
The models used thus far capture different phenomena of the experimental results of HIT-SI

<table>
<thead>
<tr>
<th>Experimental Observation</th>
<th>Zero-β MHD</th>
<th>Single Temperature MHD</th>
<th>Isothermal Electrons MHD</th>
<th>Two-Temperature MHD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain increases with $f_{inj}$</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>?</td>
</tr>
<tr>
<td>$\beta$ increases with $f_{inj}$</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>?</td>
</tr>
<tr>
<td>Outward Shift of Magnetic Axis at high-$f_{inj}$</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Increased Toroidal Symmetry of Surface fields</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>?</td>
</tr>
<tr>
<td>Increased $n = 2$ activity during relaxation at higher $f_{inj}$</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$Z_{inj} \sim f_{inj}$</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>?</td>
</tr>
</tbody>
</table>
Experimental signals serve as a guide for how well the simulations capture the physics

- A set of magnetic probes guide the bulk of the comparisons.
- There are 96 surface probes, comprising two toroidal and four poloidal arrays of 16 probes. (Green circles)
- An internal probe inserted up to the magnetic axis (R=32cm) is shown as a black line.