FRC Simulations using the NIMROD Code

R. D. Milroy, A.I.D. Macnab, C.C. Kim
Ψ-Center – University of Washington
C.R. Sovinec
Ψ-Center – University of Wisconsin, Madison

Ψ-Center Team

Outline

- Overview of Code
- FRC Acceleration and Translation
- End-Shorting and Spin-Up
- n=2 Rotational Mode

NIMROD

- Extended MHD code that includes: Hall physics, anisotropic heat conduction, non-local and particle closures for higher order kinetic effects.
- High order finite elements in two dimensions and spectral in the third dimension allows for 2D and 3D functionality.
- Implicit time stepping is used for stability beyond the CFL threshold, and for stability with the Hall term.
- Operates both linearly and nonlinearly
- Modular Fortran 90 implementation.

*C.R. Sovinec et al., J. Comp. Phys. 195, 355 (2004)

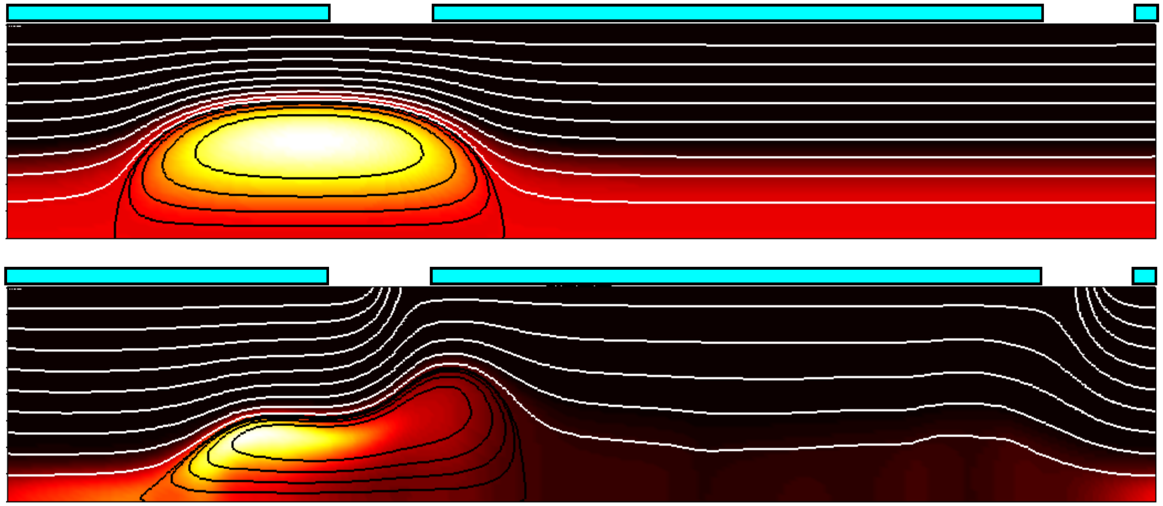
Why use NIMROD for FRC simulations

- Leverage work that has already been done, and take advantage of ongoing efforts.
- NIMROD has many features essential for accurate MHD simulations of FRCs.
  - Finite element in r-z plane and spectral in θ-direction.
  - Ability to handle anisotropic transport
  - Inclusion of Hall physics.

FRC Acceleration and Translation

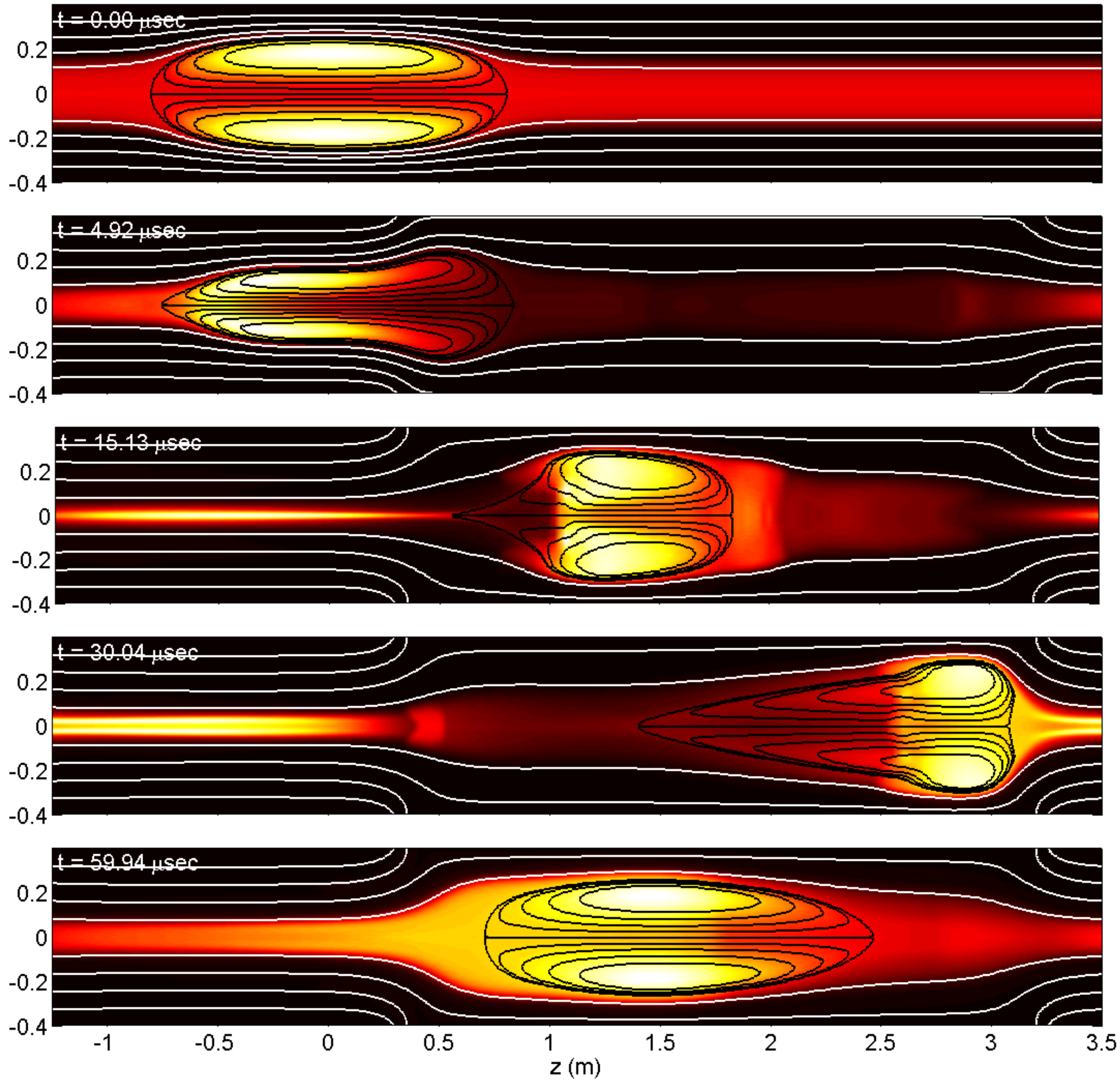
- Acceleration & translation is an important part of several Ψ-Center supported experiments (TCS, PHD, and FRX-L)
- Codes typically have difficulty translating a configuration across a numerical mesh due to diffusion associated with the convective derivative.
- Conservation of mass and magnetic flux is an important validation of a codes ability to treat translation.

Acceleration and translation simulation

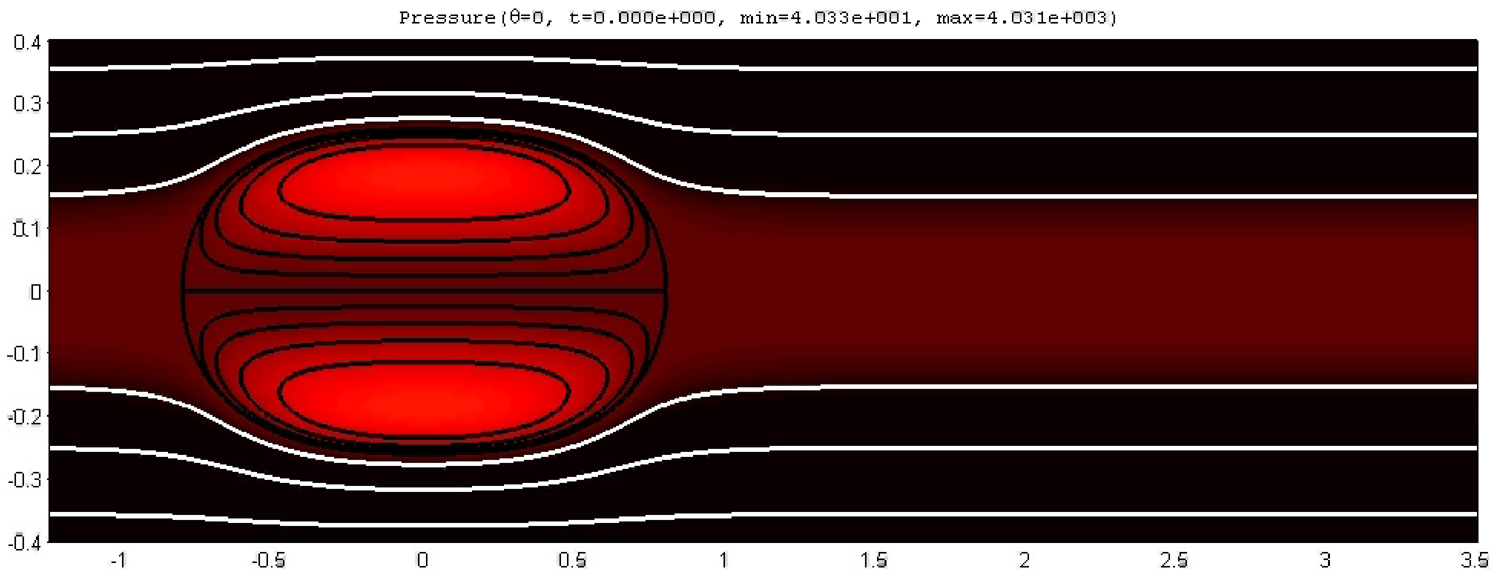


- Start with FRC equilibrium:
  - Increase flux on *coil* to left of FRC by factor of 4.5.
  - FRC accelerates to ~ 1.5 x c_s.
  - FRC bounces off mirror on RHS.
- Key Parameters
  - Reynolds number = $R_w c_s/\nu$ ~ 80
  - Lundquist number = $R_w c_s/\eta$ ~ 8000

Pressure Contours of a Translating FRC



Acceleration and translation simulation



Very Little Flux Lost by Numerical Diffusion

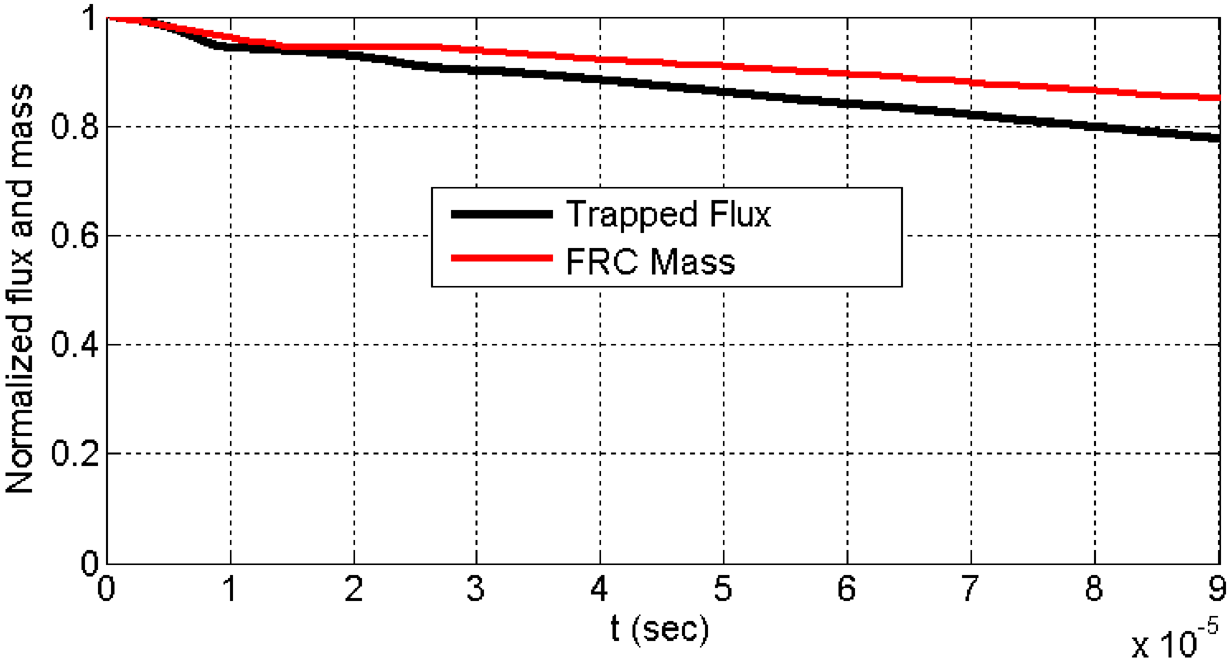
- Nimrod's finite element algorithm can simulate a translating FRC without imposing significant numerical diffusion.

Expected Lifetime

$$\tau_\phi = \frac{r_s^2}{16D} = 0.42 \text{ msec}$$

Measured Lifetime

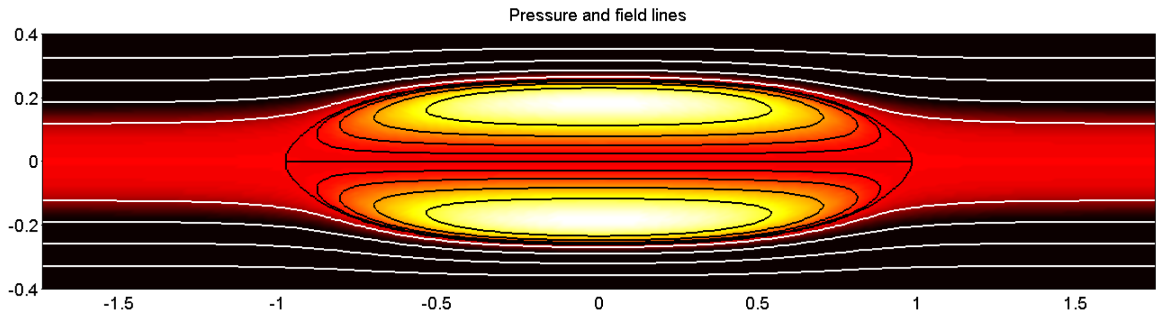
$$\tau_\phi = 0.36 \text{ msec}$$



End-shorting and FRC Spin-up

- The ions in θ-Pinch formed FRCs are observed to quickly spin-up due to two principal mechanisms:
  - Particle loss
  - End-shorting
- Particle loss mechanism not included in extended MHD.
- $\mathbf{E}_{tang}$= 0 boundary condition is applied in Nimrod.
  - This combined with the Hall (and ∇P_e) does capture end-shorting
- Observed rotation frequency characterized by $\alpha = \frac{\Omega_i}{\Omega_{di}} \sim 1$
  - For a rigid-rotor profile, with α=1, the electrons and ions each carry their own current, and the radial electric field is ~ 0.

End-shorting Mechanism



- For simplicity, assume $\nabla P_e = 0$ (cold electrons)
  - Magnetic field convects with the electron fluid.
  - Open field-line electrons rotate faster at midplane, where dB_z/dr, and hence J_θ is largest.
    - » Field lines develop a θ-component, and "wind-up".
    - » The resulting JxB force leads to a net torque in the θ-direction
- Inclusion of the ∇P_e term complicates the picture a bit, but the basic mechanism is unchanged.

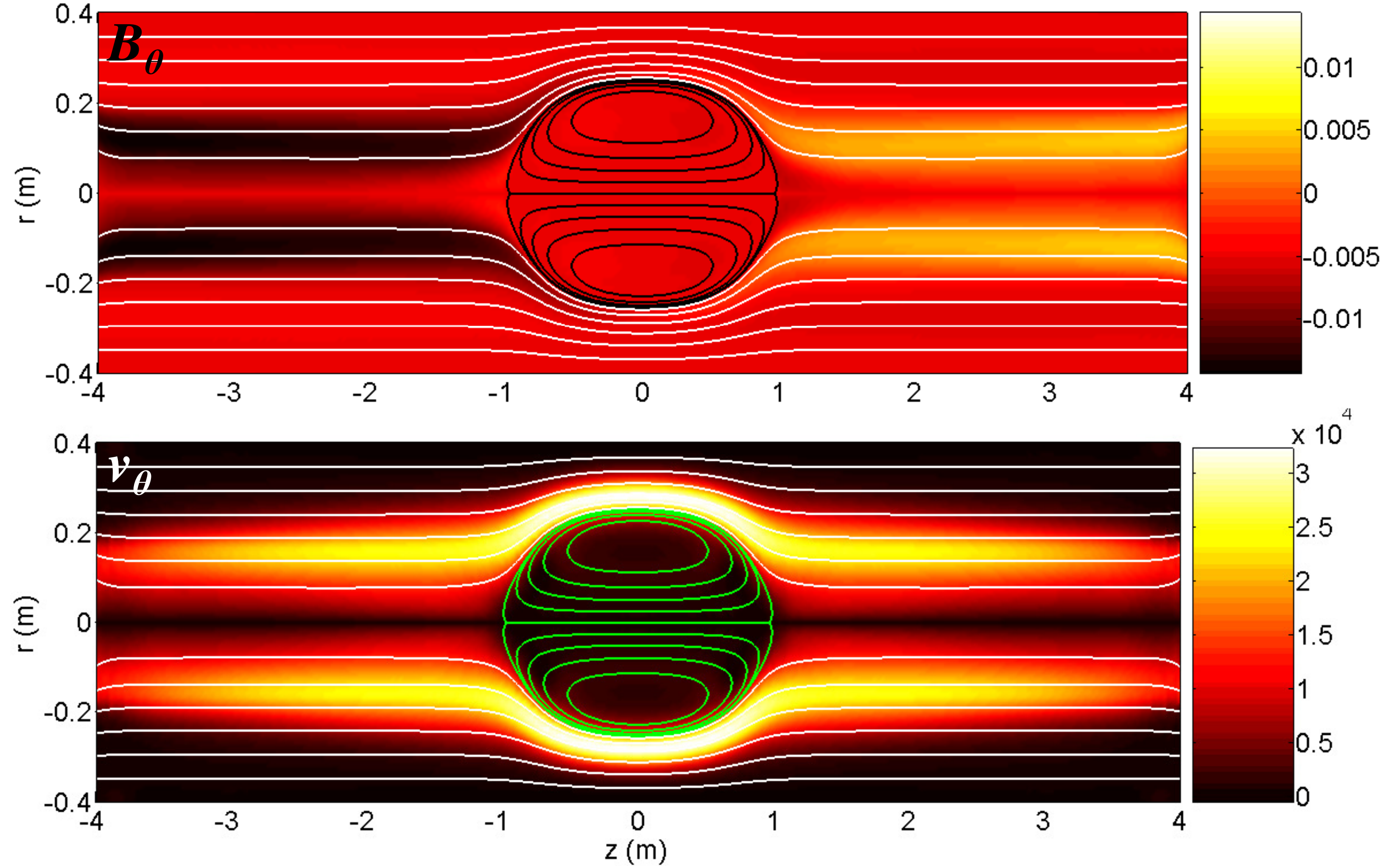
End-shorting Simulation

- End-shorting is simulated in Nimrod
  - Start with MHD equilibrium
  - Apply $E_{tang} = 0$ boundary condition
  - Include Hall and ∇P_e terms

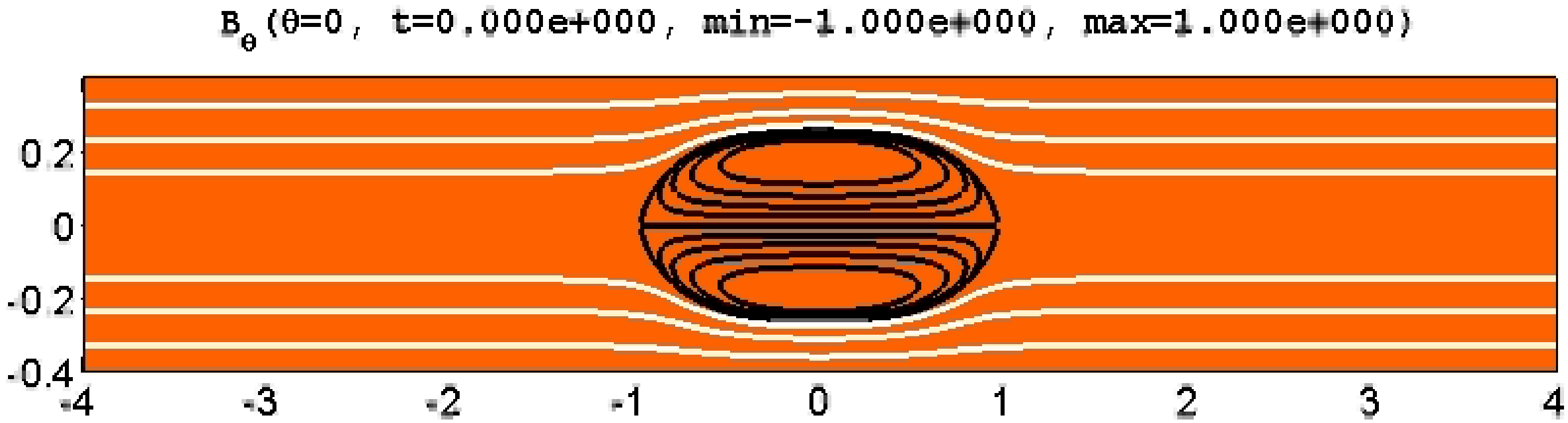
- A toroidal field is induced at the ends:
  This leads to a $(J_r \times B_z)$ torque on the plasma.
- Simulation Parameters:
  - n_o=1.25x10^{20} m^{-3}, T_e = 50 eV, T_i = 150 eV, B_{ext} = 0.1 T, R_{wall}=0.4, x_s = 0.65, r_s = 0.26, Ω_{di}=1.09x10^5, V_A = 1.4x10^5, τ_A= $L_{1/2}/V_A$ = 25 μsec.

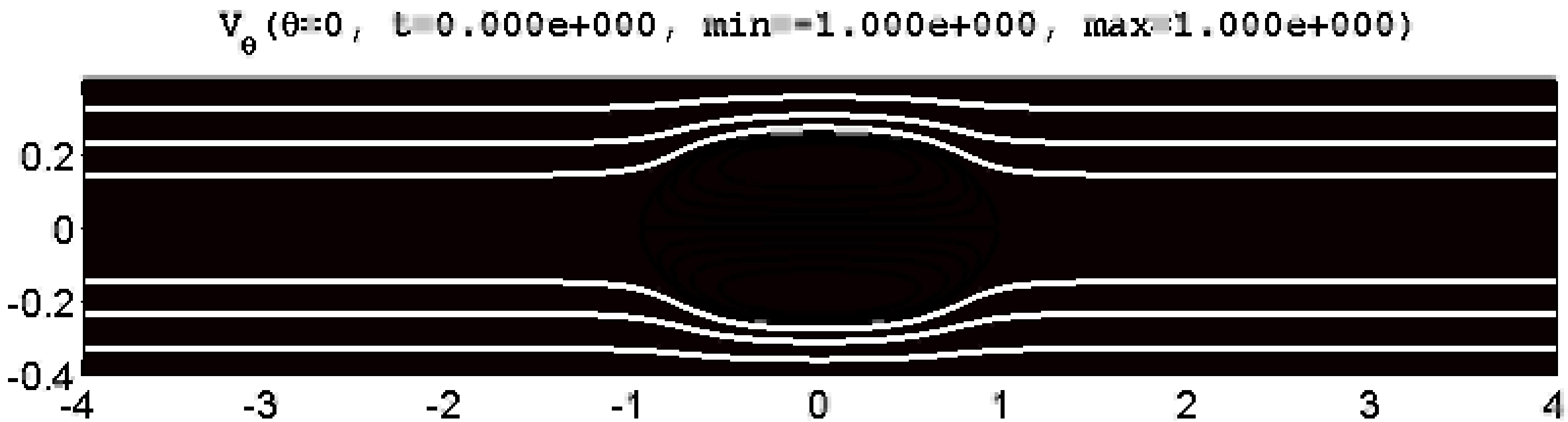
An Induced B_θ Applies a Torque to the Open Field Line Plasma

Induced toroidal field and toroidal velocity at 50 μsec.



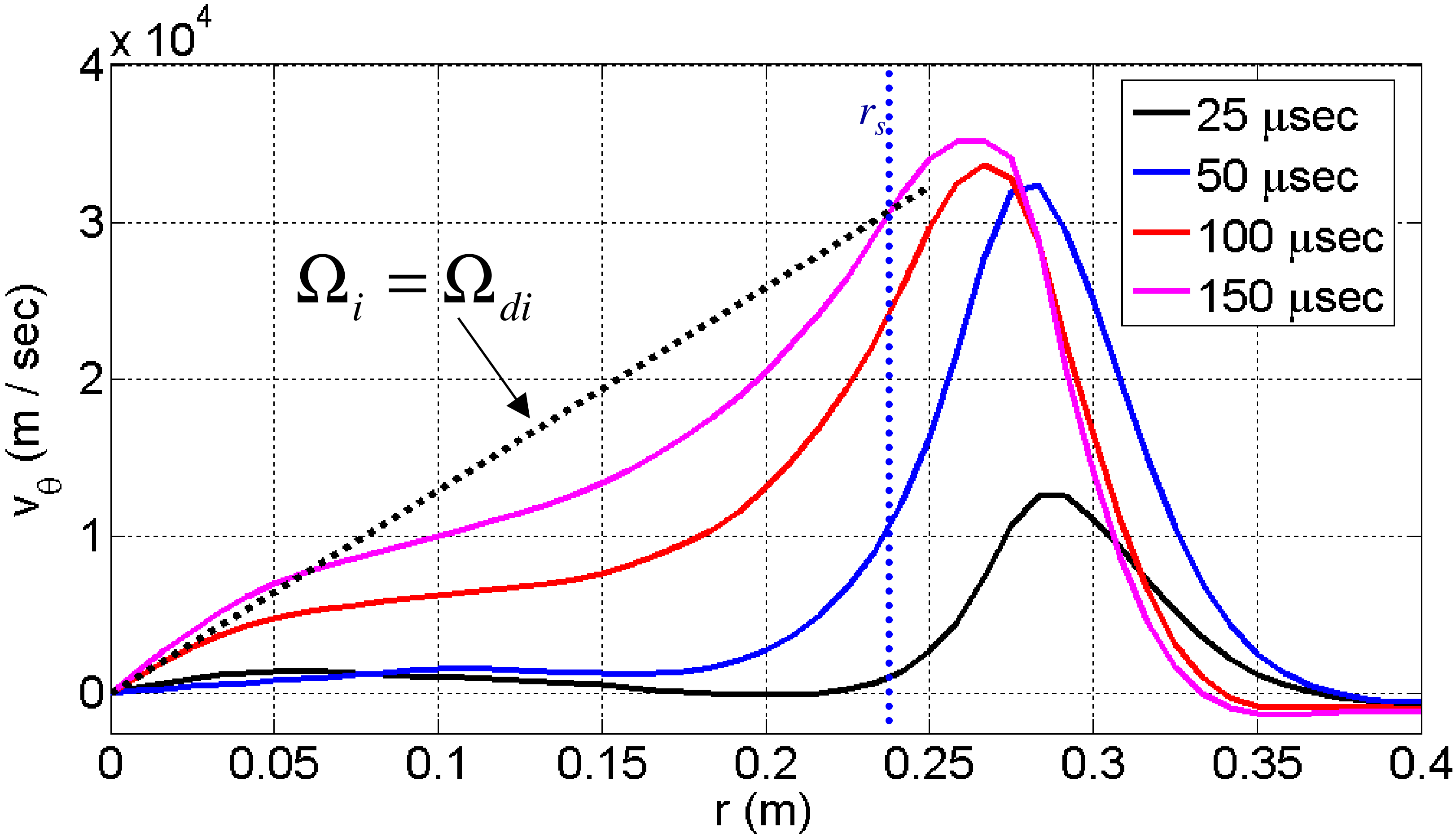
B_θ and V_θ during spin-up





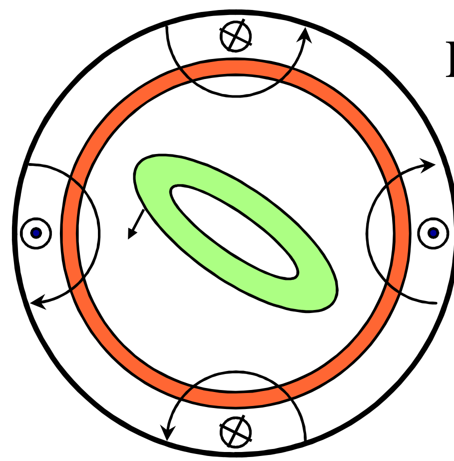
End-shorting and FRC Spin-up

- Open field line plasma quickly spins up to Ω_{di}.
- As rotation *diffuses* in, the inner field lines spin up too.



Background: n=2 Rotational Instability

- Has always been observed in FRCs and θ-pinches
  - Driven by centrifugal forces in a rotating plasma
  - It has been stabilized by weak multipoles with $B_m^2/2\mu_o >$ centrifugal pressure
  - We are attempting to duplicate recent experimental observations [H.Y. Guo, et al., Phys. Rev. Lett. 95, 175001 (2005)] and test their theory that a small toroidal field can stabilize this mode.

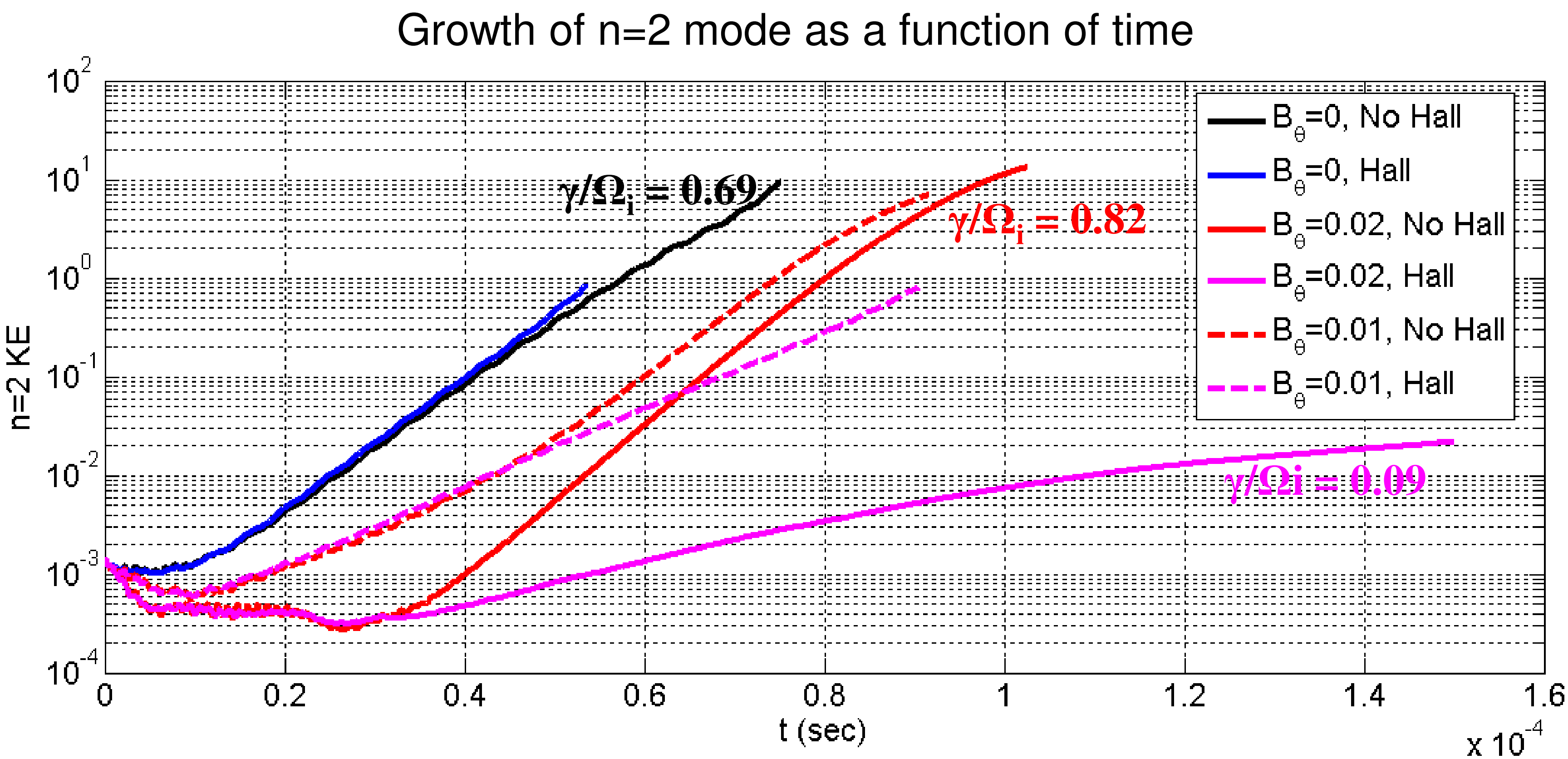


Rotational Instability Study Method

- Start with non-rotating equilibrium, add a rigid rotation and toroidal magnetic field to the n=0 component.
  - Let NIMROD seek a new equilibrium
- Running MHD, with and without the Hall term
  - With and without toroidal fields
  - Rotation rate ~ $\Omega_\mathrm{i} = \Omega_\mathrm{di} = 4KT_i / (R^2 B)$
- Initialize with n=2 perturbation in u_r the FRC
- Add a toroidal field $rB_\theta \sim \psi/\psi_o$ inside the FRC and zero outside.

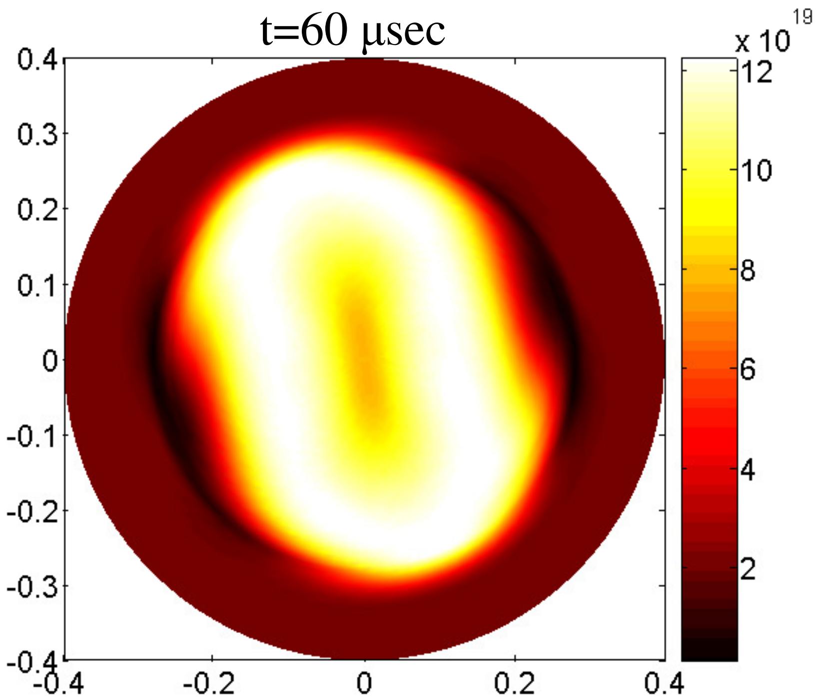
Growth of n=2 mode with and without toroidal field

- With toroidal field, growth is delayed as initial perturbation is not aligned with fastest mode.
  - *The Hall term dramatically reduces the growth of this mode.*

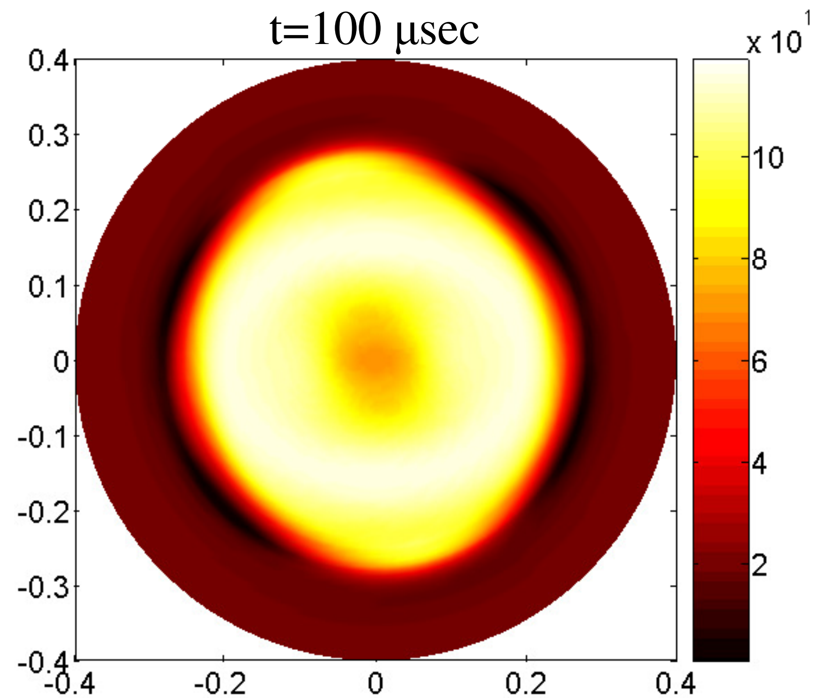


Density profiles at axial midplane

No toroidal field

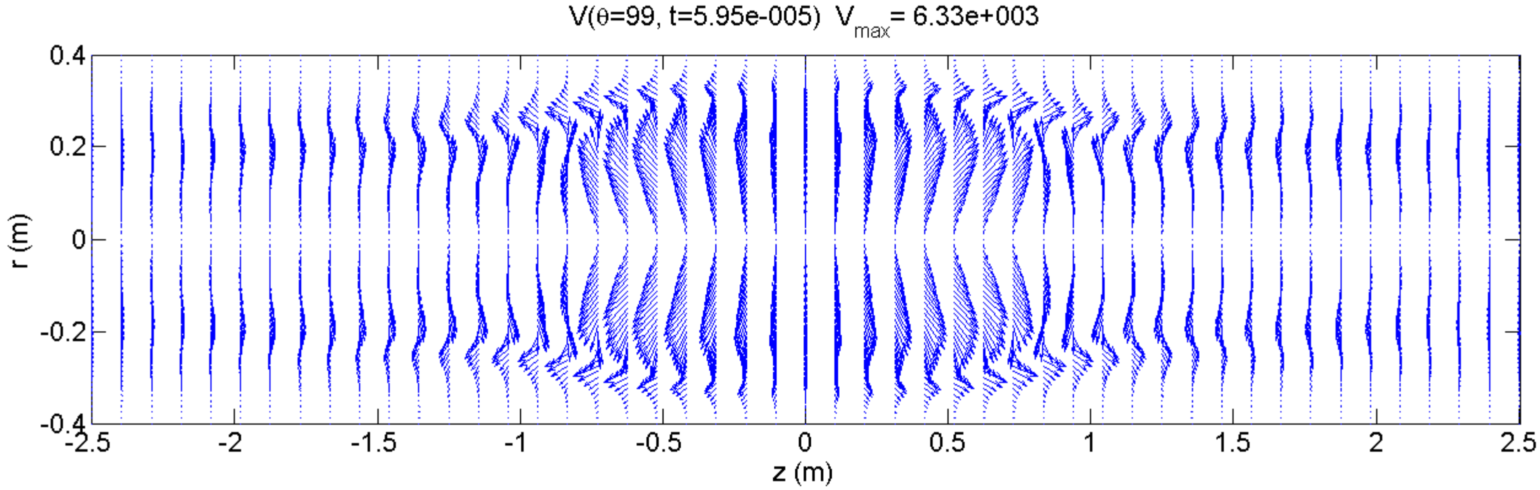


With toroidal field

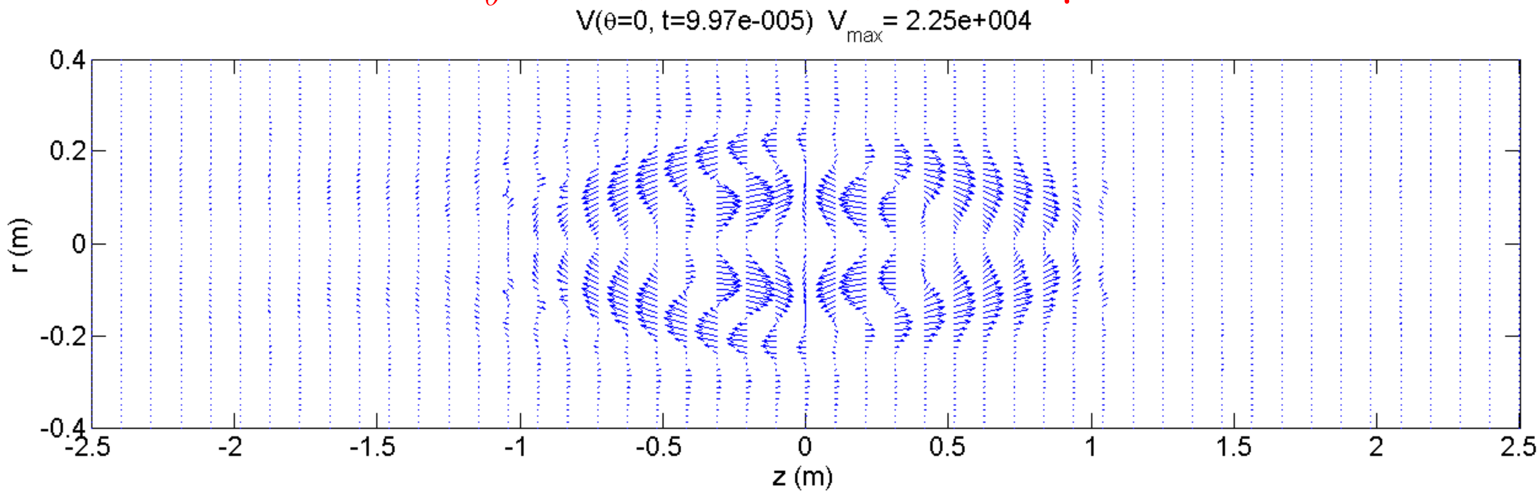


Velocity Vectors: n=2 Component in Plane of Maximum Perturbation

B_θ=0.0, No Hall term, t=60 μsec



B_θ=0.02T, No Hall term, t=100 μsec



Conclusions

- The NIMROD code is a valuable tool for research of Emerging Concept (EC) experiments.
  - Includes a robust implementation of Hall physics, anisotropic transport, and an ability to run 2D and 3D.
  - Can continue to leverage from improvement made by the entire NIMROD team.
- We have demonstrated this capability with simulations of:
  - FRC acceleration and translation
  - FRC spin-up due to end-shorting
  - Potential stabilization of the n=2 rotational instability with a small toroidal magnetic field.