MH4D Development

Eric Meier
Plasma Science and Innovation Center
University of Washington

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Outline

- Summary of previous work
- New b.c. development
  - Non-parallel periodic boundaries
  - Operator matrix b.c.
- Atomic physics development
  - Spheromak simulations
- Other physics development
  - Variable resistivity, ohmic heating
- Benchmarking
  - Screw pinch instability
  - ZaP simulations
- Future work

Summary of previous work

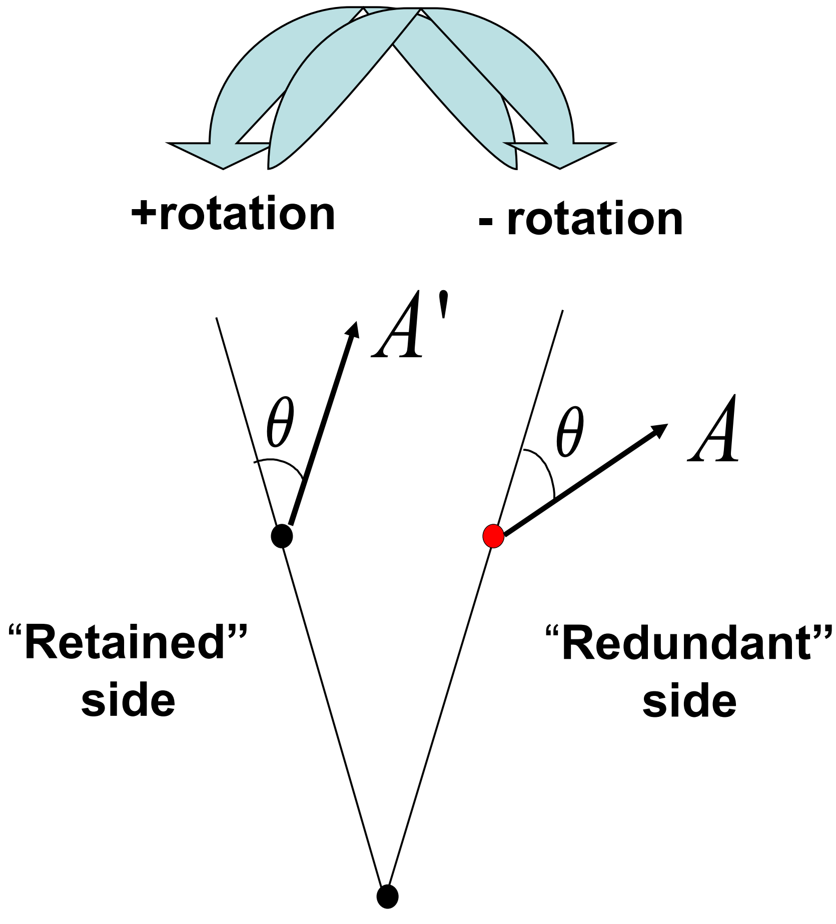
Previous Work

- Insulating b.c.
  - Apply magnetic field b.c.
  - Allow non-zero $E_{tang.;}$ $E_{normal}=0$
  - Allow v_{normal} (density floor required)
- Periodic b.c.
  - Multi-directional
- Initial ZaP simulations

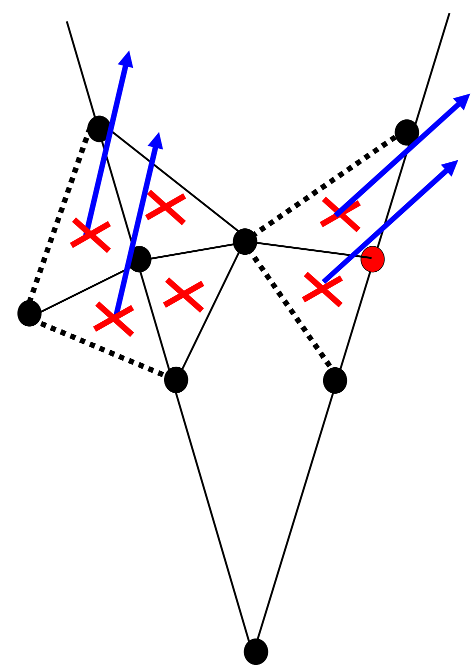
New boundary condition development

Non-parallel periodic b.c.

Rotation required across boundary



Curl computation uses vector information from surrounding tetrahedra



Non-parallel periodic b.c.

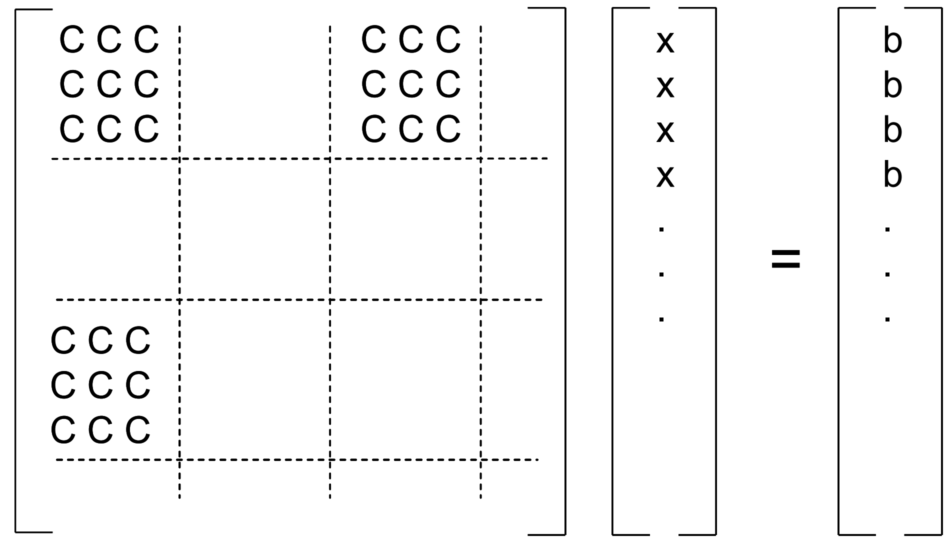
Operator matrices must be modified

Normally: $Cx = b$

For rotated vectors x' **and** b'**:**

$$CR^T x' = R^T b' \;\Rightarrow\; RCR^T x' = b'$$

$$\Rightarrow C' = RCR^T$$



For boundary rotation and periodic rotation:

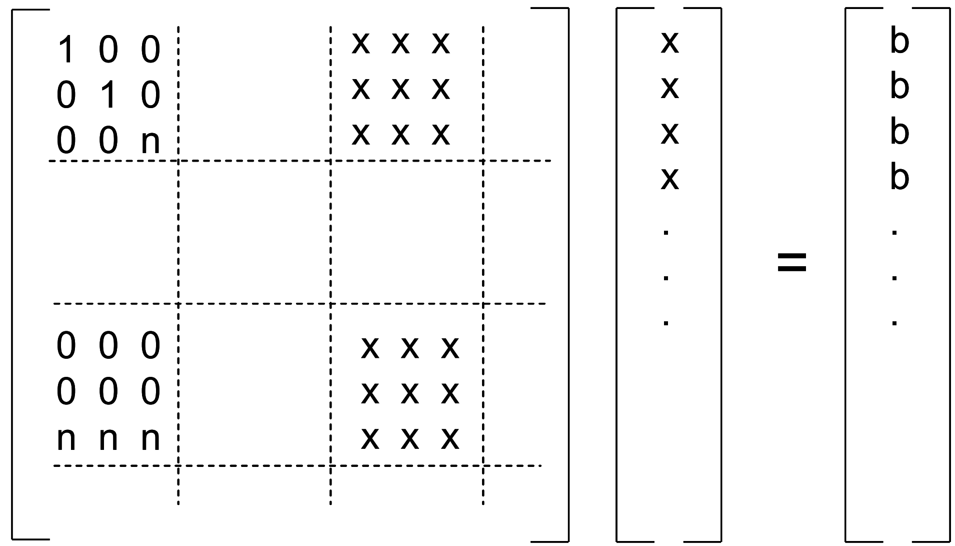
$$CR_p{}^T R_b{}^T x' = R_p{}^T R_b{}^T b' \;\Rightarrow\; R_b R_p C R_b{}^T R_p{}^T x' = b'$$

$$\Rightarrow C' = R_b R_p C R_p{}^T R_b{}^T$$

Operator matrix b.c.

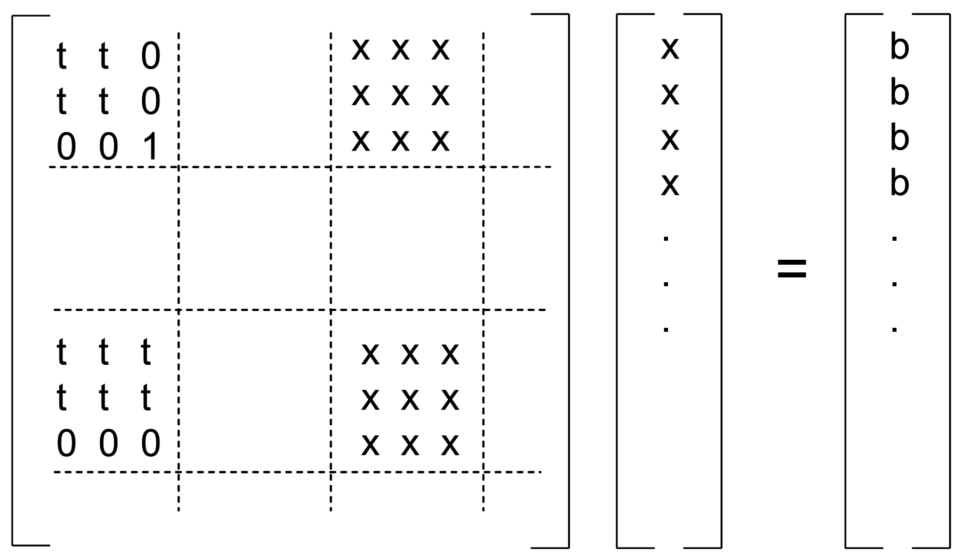
Conducting boundary point →

- Self-coupling is identity for tangential components.
- Neighbors couple only to normal component.



Insulating boundary point →

- Self-coupling is identity for normal components.
- Neighbors couple only to tangential components.



Atomic physics development

Atomic Physics Development

- Phase I
  - Modify continuity equations
    - Assume constant neutral background density
    - Allow variable neutral background density
- Phase II
  - Modify momentum and energy equations

Atomic Physics Development
Phase I

Temperature dependent ionization

$$\frac{\partial \rho_i}{\partial t} = -\nabla \, (\rho_i \mathbf{v}_i) + \Gamma_{ion} \quad \frac{\text{kg}}{\text{m}^3}\,\text{s}^{-1}$$

$$\Gamma_i = \langle \sigma_{ion} \mathbf{v}_e \rangle \rho_i n_n$$

Use $\langle \sigma_{ion} \mathbf{v}_e \rangle = \dfrac{2x10^{-13}}{6.0 + T_{e,eV}/13.6} \left(\dfrac{T_{e,eV}}{13.6} \right)^{1/2} \exp\left(-\dfrac{13.6}{T_{e,eV}} \right) \text{m}^3\text{s}^{-1}$

(ref. Goldston, Rutherford, Intro. to Plasma Physics, 1995, p. 151)

If $\mathbf{v_i}$=0, analytically: $\rho_i = \rho_{i,0} \exp\left(\langle \sigma_{ion} \mathbf{v}_e \rangle n_n t_{total} \right)$

Atomic Physics Development
Phase I

Temperature dependent recombination

$$\frac{\partial \rho_i}{\partial t} = -\nabla\ (\rho_i \mathbf{v}_e) - \Gamma_{rec}$$

$$\Gamma_{rec} = \langle \sigma_{rec} \mathbf{v}_e \rangle \rho_i n_i$$

$$= \langle \sigma_{rec} \mathbf{v}_e \rangle \rho_i^2 \frac{1}{1.67x10^{-27}}$$

Use approximation: $\langle \sigma \mathbf{v}_e \rangle = 0.7x10^{-19} \left(\frac{13.6}{T_{e,eV}} \right)^{1/2}$ $\mathrm{m^3 s^{-1}}$

(ref. Goldston, Rutherford, Intro. to Plasma Physics, 1995, p. 152)

If $\mathbf{v_i}$=0, analytically: $\rho_i = \dfrac{1}{t_{total} \langle \sigma_{rec} \mathbf{v}_e \rangle \frac{1}{1.67x10^{-27}} + \frac{1}{\rho_0}}$

Atomic Physics Development
Phase I

Track neutral fluid density

Assuming **v**=0,

$$\frac{\partial \rho_n}{\partial t} = -\Gamma_{ion} + \Gamma_{rec} \; ; \; \frac{\partial \rho_i}{\partial t} = \Gamma_{ion} - \Gamma_{rec}$$

Compute Γ_{ion} and Γ_{rec} in each cell. Must not violate simple accounting!

- if $\Gamma_{ion} dt$ is greater than ρ_{ion} , $\rho_{ion} = \rho_{ion} + \rho_n$; $\rho_n = 0$
- if $\Gamma_{rec} dt$ is greater than $\rho_i - \rho_{i,floor}$, $\rho_n = \rho_n + \left(\rho_i - \rho_{i,floor} \right)$;

  $\rho_i = \rho_{i,floor}$

Atomic Physics Development
Phase I

Comments

- MH4D gives precisely the expected analytical solutions for simple problems.
- In coronal equilibrium, MH4D predicts the expected ~50% ionization at 1.5 eV.
- Predicts the expected ~100% ionization at 13.6 eV.

Atomic Physics Development
Phase II

Single-fluid plasma model

- **Cold neutral fluid with initial distribution**
- **Assume** v_n**=0**

$$\frac{\partial \rho_i}{\partial t} = -\nabla \ (\rho_i \mathbf{v}_i) + \Gamma_{ion} - \Gamma_{rec}$$

$$\frac{\partial \rho_n}{\partial t} = -\Gamma_{ion} + \Gamma_{rec}$$

$$\rho_i \frac{\partial \mathbf{v}_i}{\partial t} + \nabla(p_e + p_i) = \mathbf{j} \times \mathbf{B} - \left(\Gamma_{rec} + \Gamma_{cx} \right) m_i \mathbf{v}_i$$

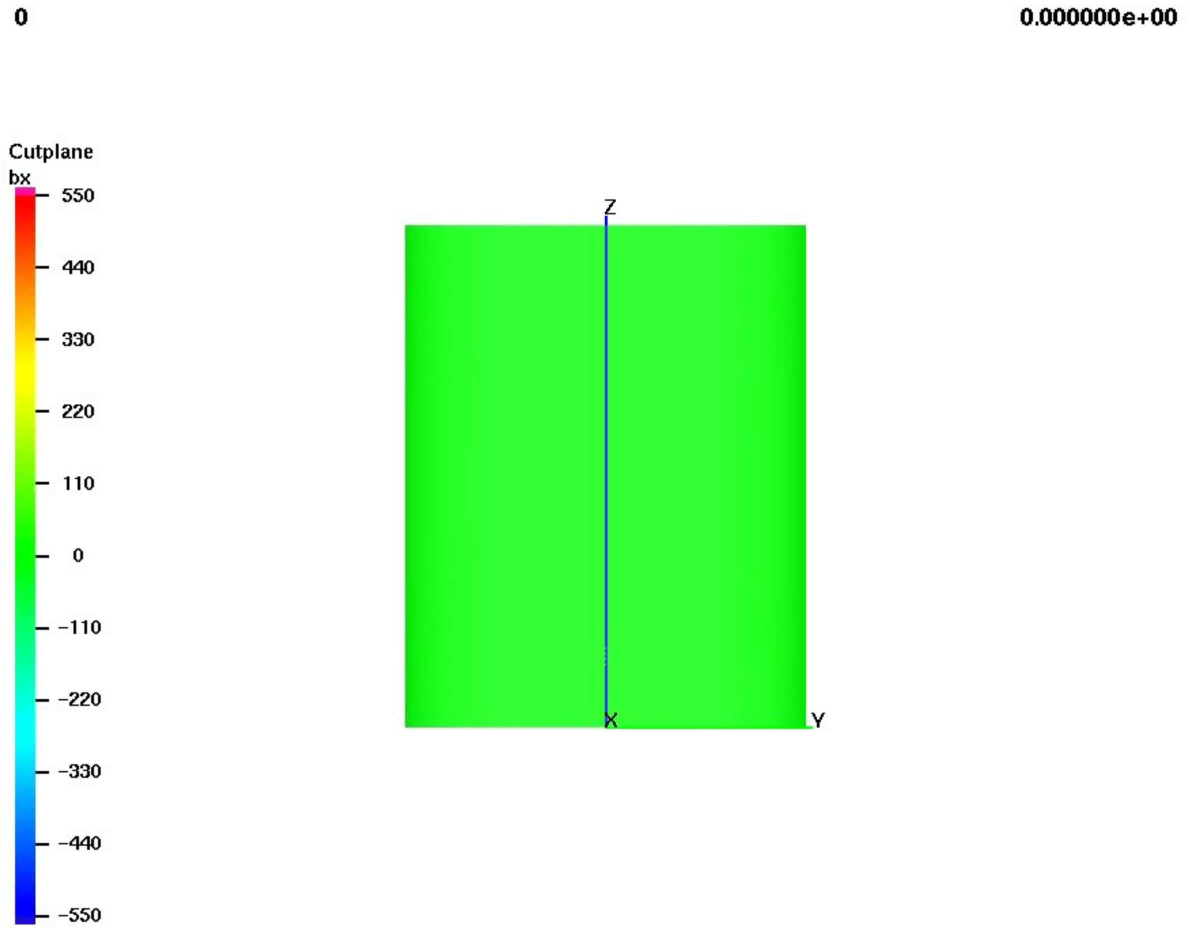
$$\frac{\partial p}{\partial t} + \mathbf{v} \ \nabla p = -\gamma p \nabla \ \mathbf{v} + \left(\gamma - 1 \right) \eta \mathbf{j}^2 - \left(\gamma - 1 \right) \left(\Gamma_{ion} + \Gamma_{cx} \right) m_n \Phi_{ion}$$
$$- \left(\Gamma_{rec} + \Gamma_{cx} \right) m_i p$$

Atomic Physics Development
Phase II

- Spheromak tilt problem
  - "Tuna can" flux conserver
  - L/R=2.5
  - Beta=2.5%
  - Flat pressure profile
  - Force–free i.c.→ **B**
    - Find **A** that satisfies curl(**A**)=**B**
    - Allow $\mathbf{A}_{tangential}$, on boundaries, but d/dt($A_{tang.)}$.=0.
  - Find tilt growth rate with and without atomic physics

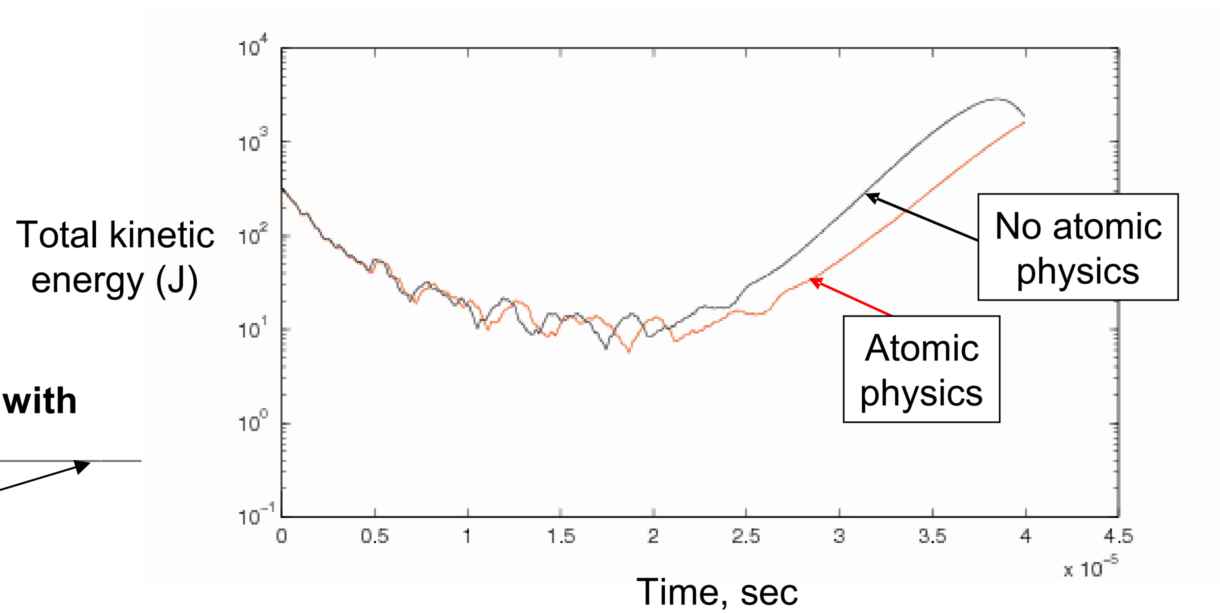
Atomic Physics Development
Phase II

Movie

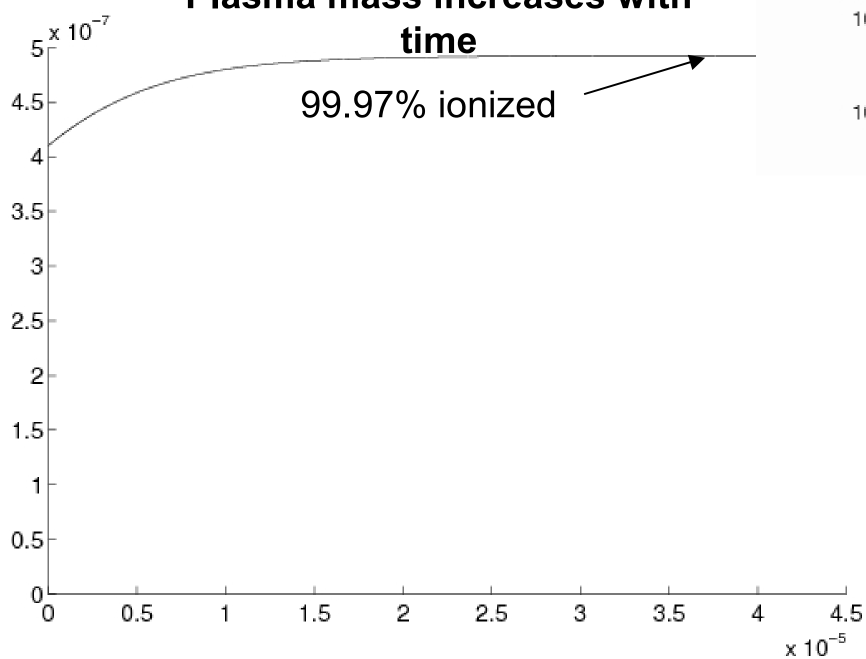


Atomic Physics Development
Phase II

K.E. growth with and without atomic physics



Plasma mass increases with time



Tilt growth rate is 14.0% less with atomic physics.

$$\gamma\tau_A = 0.047 \text{ vs. } \gamma\tau_A = 0.055$$

Other physics development

Variable resistivity, Ohmic heating

- Chodura (anomalous) resistivity:

$$\eta_c = \upsilon_c \frac{m_e}{ne^2} \; , \; \upsilon_c = C_c \omega_{pi} \left[1 - \exp\left(-\frac{v_e}{f v_s} \right) \right] \; , \; v_e = \frac{j}{ne} \; , \; v_s = \sqrt{\frac{\gamma p}{\rho}}$$

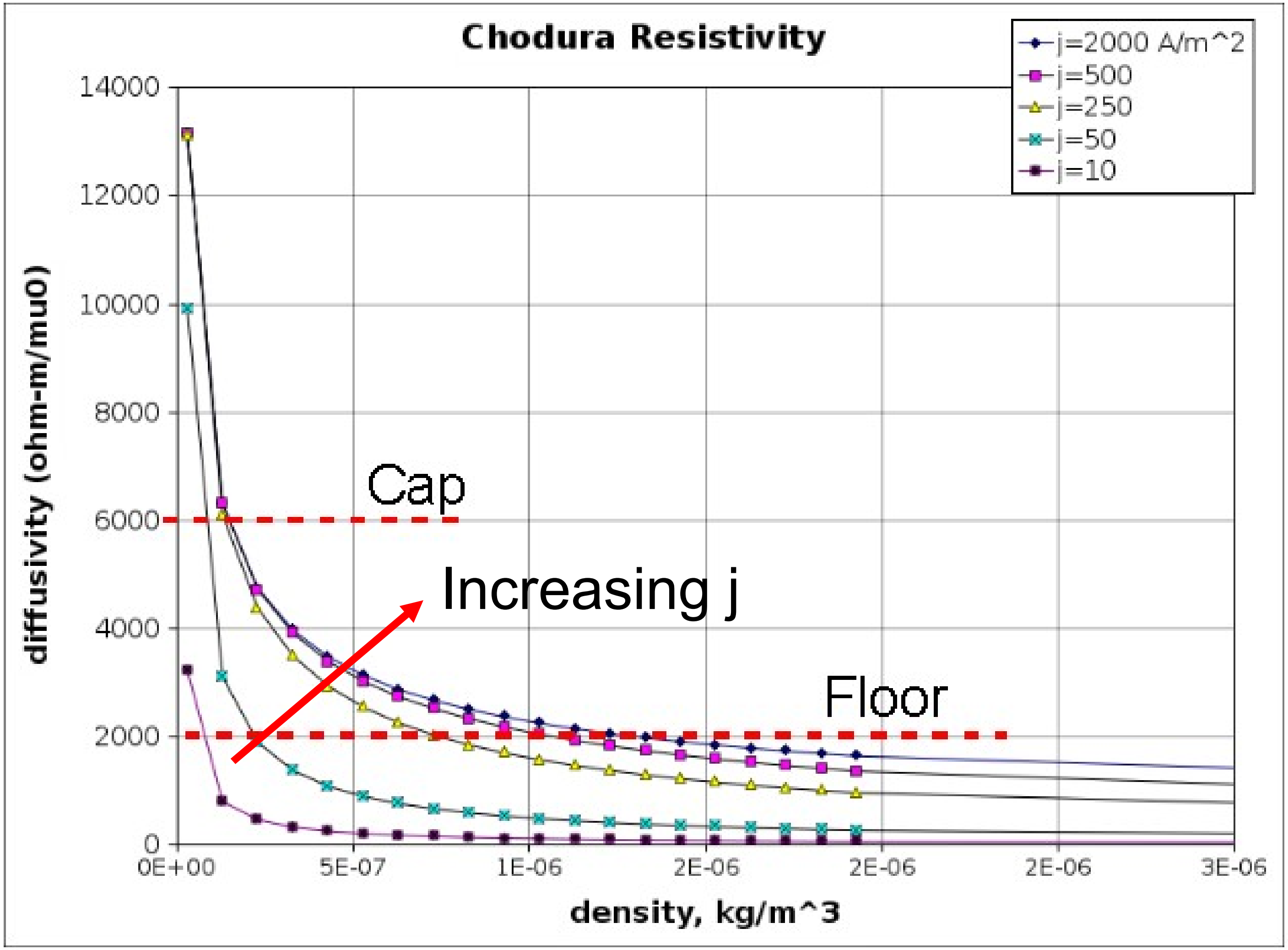
$$C_c \approx 1 \; , f \approx 3$$

- Spitzer resistivity: $\eta_{Sp.} = \dfrac{\left(5x10^{-5}\right) \ln \Lambda}{T_{eV}^{\frac{3}{2}}}$

- Explicit timestep limit: $2 \dfrac{\eta}{\mu_0} \dfrac{\Delta t}{\Delta x^2} < 1$

- Ohmic heating: $\dfrac{\partial p}{\partial t} + \mathbf{v} \;\; \nabla p = -\gamma p \nabla \;\; \mathbf{v} + \left(\gamma - 1 \right) \eta \mathbf{j}^2$

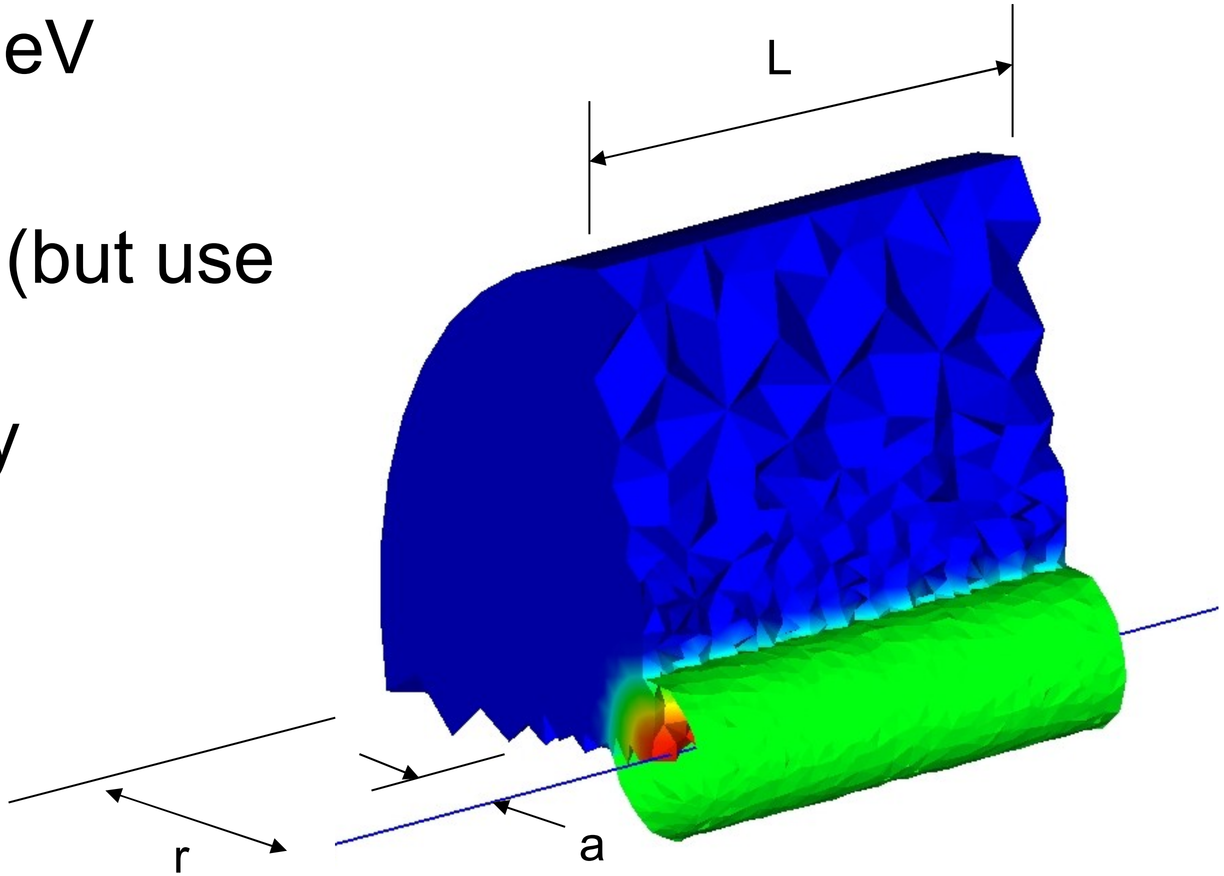
Chodura resistivity



Benchmarking

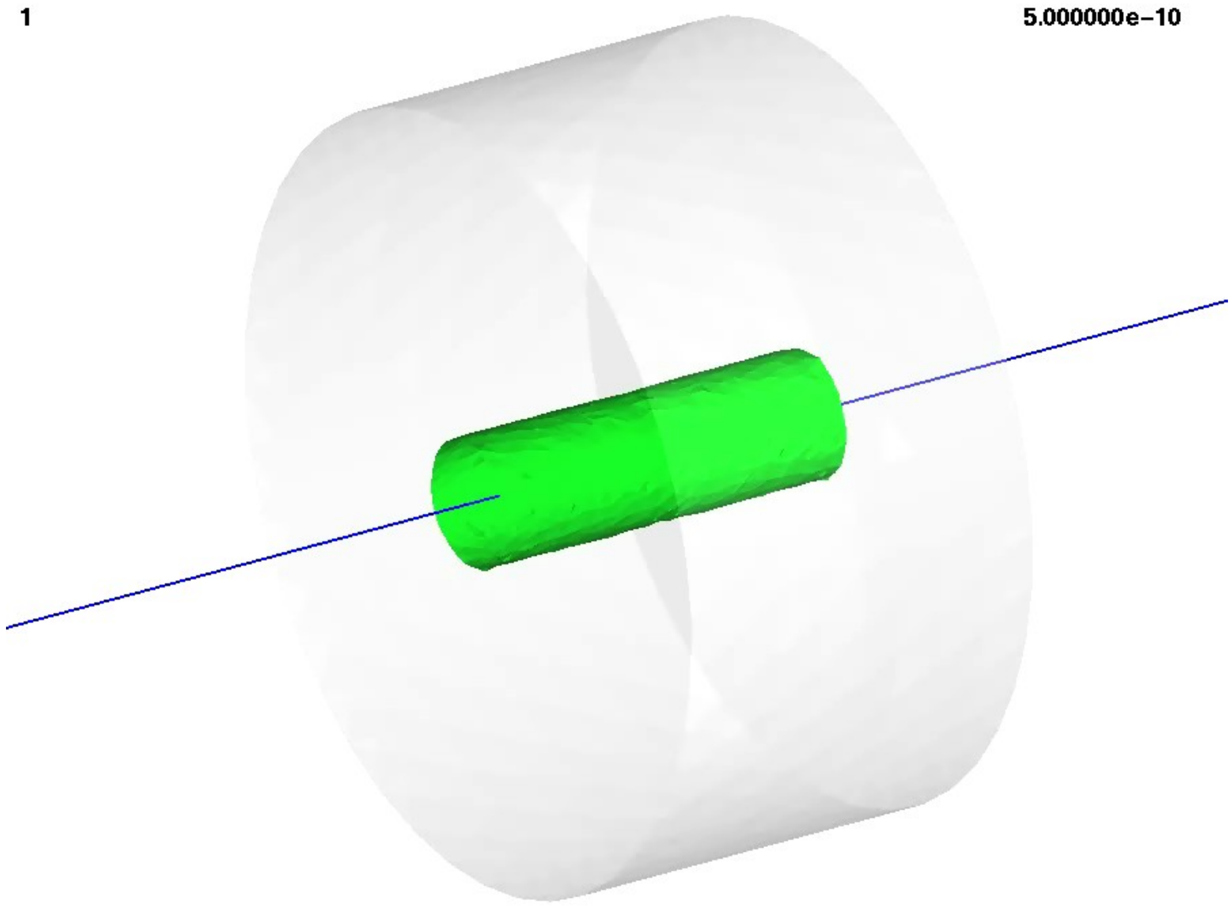
Screw pinch benchmark

- Parabolic pressure profile
- T=const.=100 eV
- B_z=const.
- Density ~p/kT (but use density floor)
- Helical velocity perturbation

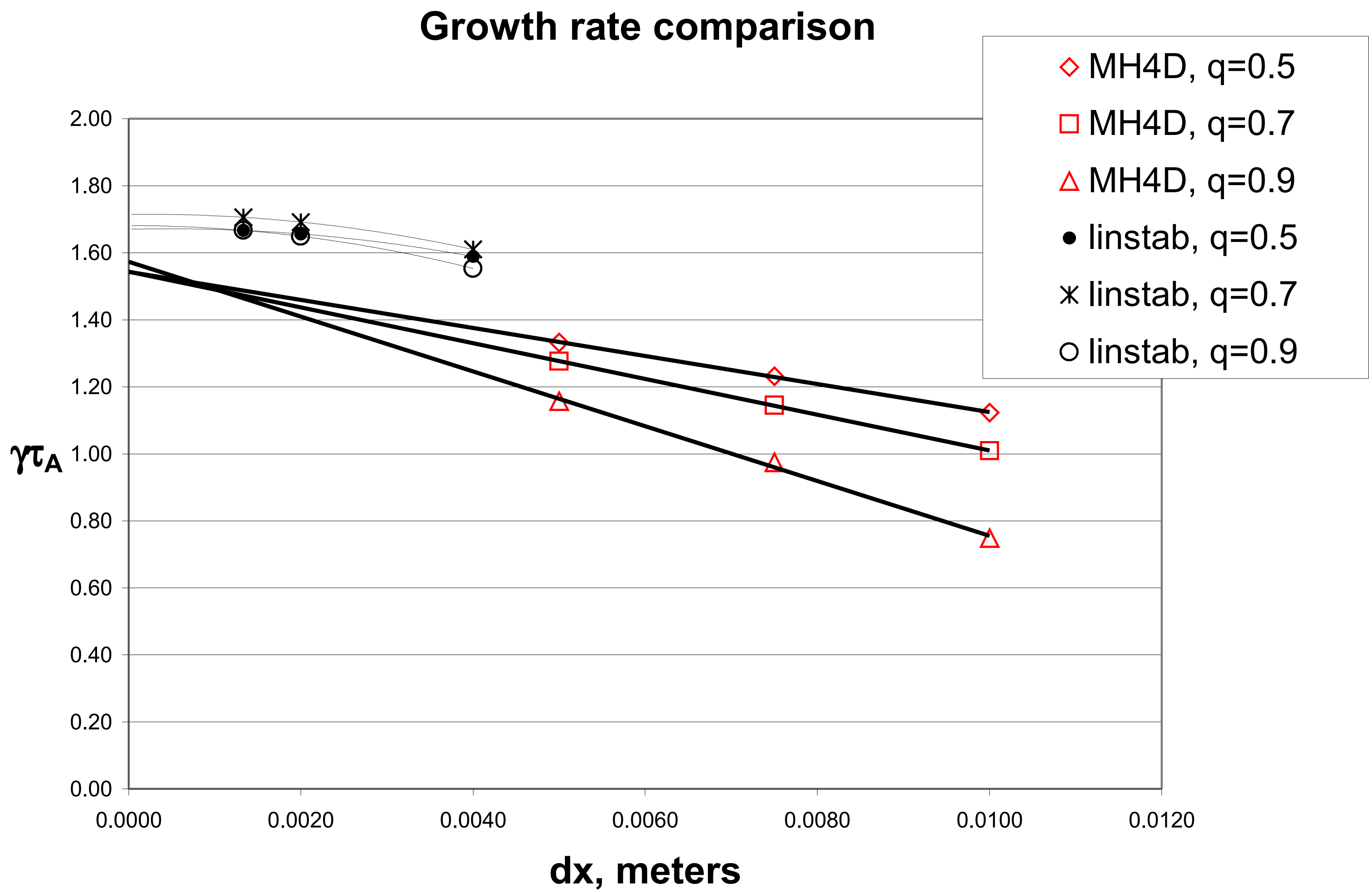


Screw pinch benchmark, cont.'d

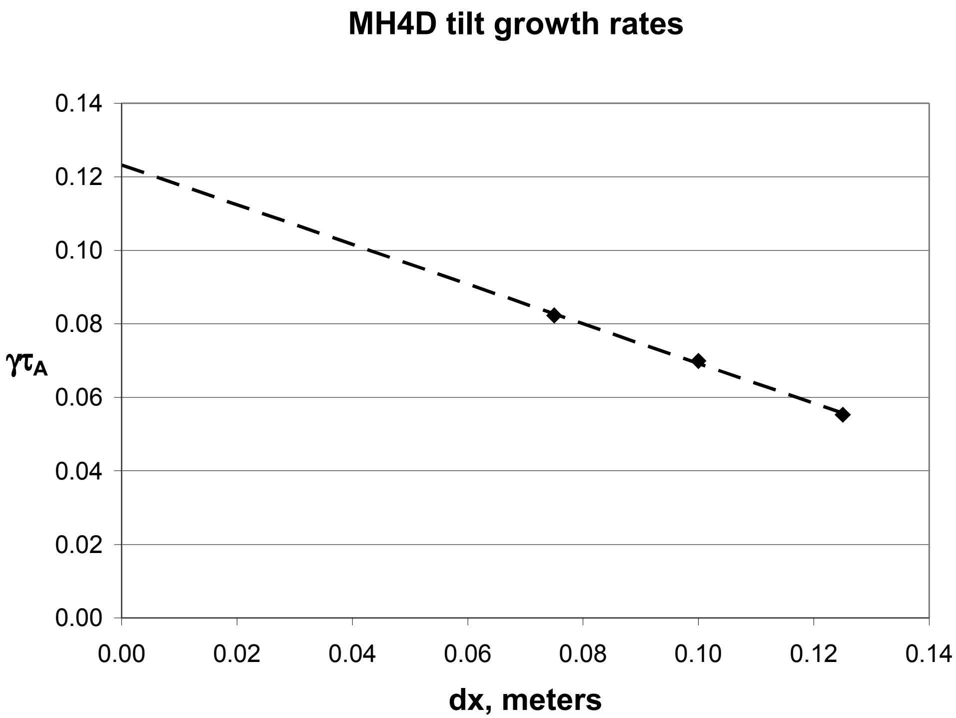
Movie



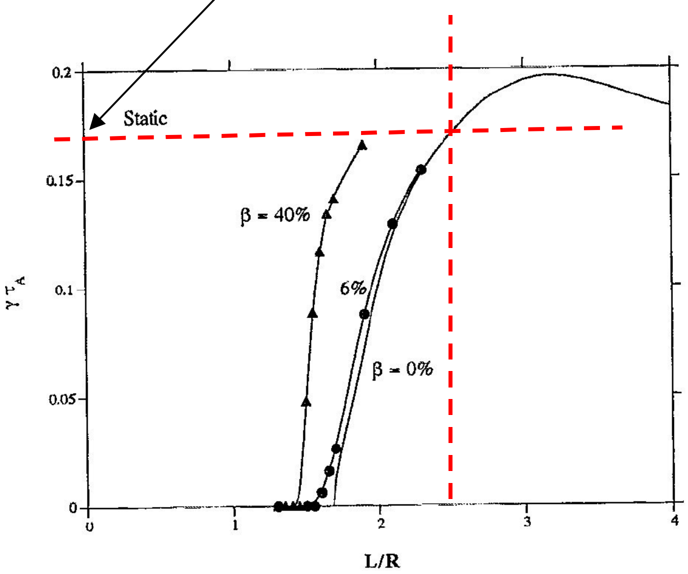
Screw pinch benchmark



Spheromak benchmark



Published data*
indicates 0.17



* Shumlak, Fowler, Morse, Phys. Plasmas, Vol. 1, No. 3, March 1994

Comparison to MACH2 via ZaP simulations

- Work in progress
- Resolution unlikely to approach MACH2
- Interplay of several factors is complicating matters:
  - Significant resistivity range is needed
  - High resolution is necessary
  - Low density floor would be ideal to capture low density current paths in plasma

Future work

Future work

- To finish MS:
  - Implement implicit and semi-implicit methods for high resolution sim's?
  - Atomic physics in ZaP
  - Significant time available on Bassi
- PhD work:
  - To be discussed shortly by Dr. Shumlak