The Hybrid Kinetic-MHD Equations[a]

- in the limit $n_h \ll n_0$, $\beta_h \sim \beta_0$, quasi neutrality, only modification of MHD equations is addition of the hot particle pressure tensor in the momentum equation:

$$\rho\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U}\right) = \mathbf{J}\times\mathbf{B} - \nabla p_b - \nabla\cdot\underline{\mathbf{p}}_h$$

  the subscripts b, h denote the bulk plasma and hot particles

- the steady state equation

$$\mathbf{J}_0\times\mathbf{B}_0 = \nabla p_0 = \nabla p_{b0} + \nabla p_{h0}$$

- evolved momentum equation is $(\mathbf{U}_s = 0)$

$$\rho_s\frac{\partial\delta\mathbf{U}}{\partial t} = \mathbf{J}_s\times\delta\mathbf{B} + \delta\mathbf{J}\times\mathbf{B}_s - \nabla\cdot\delta\underline{\mathbf{p}}_b - \nabla\cdot\delta\underline{\mathbf{p}}_h$$

[a]C.Z.Cheng,'A Kinetic MHD Model for Low Frequency Phenomena',*J. Geophys. Rev* **96**, 1991

Deposition of $\delta\underline{\mathbf{p}}_h$ onto Finite Element grid

- assume CGL-like form $\delta\underline{\mathbf{p}}_h = \begin{pmatrix} \delta p_\perp & 0 & 0 \\ 0 & \delta p_\perp & 0 \\ 0 & 0 & \delta p_\parallel \end{pmatrix}$

- evaluate pressure moment at a position $\mathbf{x}$ is

$$\begin{aligned} \delta p_\perp(\mathbf{x}) &= \int \frac{1}{2} m v_\perp^2 \delta f(\mathbf{x}, \mathbf{v}) d^3 v \\ &= \sum_{i=1}^{N} \frac{1}{2} m v_{i\perp}^2 g_0 w_i \delta^3(x - x_i) \end{aligned}$$

where sum is over the particles, m mass of the particle, $g_0 w_i$ is the perturbed phase density

The δf PIC method [a] [b]

- PIC is a Lagrangian simulation of phase space $f(\mathbf{x}, \mathbf{v})$
- PIC evolves the $f(\mathbf{x}(\mathbf{t}), \mathbf{v}(\mathbf{t}))$
- spatial grid is not inherantly necessary, but very convenient!
- in principle, $f(\mathbf{x}(\mathbf{t}), \mathbf{v}(\mathbf{t}))$ contains everything
- typically PIC is noisy, can't beat $1/\sqrt{N}$
- δf PIC reduces the discrete particle noise associated with conventional PIC
- Vlasov Equation

$$\frac{\partial f(\mathbf{z})}{\partial t} + \dot{\mathbf{z}} \cdot \frac{\partial f(\mathbf{z})}{\partial \mathbf{z}} = 0$$

$\mathbf{z}$ is the phase coordinate

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[a] S. E. Parker and W. W. Lee, 'A fully nonlinear characteristic method for gyro-kinetic simulation', *Physics of Fluids B*, **5**, 1993

[b] G. Hu and J. A. Krommes, "Generalized weighting scheme for δf particle simulation method", *Physics of Plasmas*, **1**, 1994

- split phase space distribution into steady state and evolving perturbation:

$$f = f_{eq}(\mathbf{z}) + \delta f(\mathbf{z}, t)$$

- δf evolves along the characteristics $\dot{\mathbf{z}}$ (control variates MC[c])

$$\dot{\delta f} = -\tilde{\mathbf{z}} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}}$$

using $\mathbf{z} = \mathbf{z}_{eq} + \tilde{\mathbf{z}}$ and $\dot{\mathbf{z}}_{eq} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}} = 0$

- the drift kinetic equations of motion are used as the particle characteristics

$$\dot{\mathbf{x}} = v_\parallel \hat{\mathbf{b}} + \frac{m}{eB^4}\left(u^2 + \frac{v_\perp^2}{2}\right)\left(\mathbf{B} \times \nabla \frac{B^2}{2}\right) + \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{\mu_0 m v_\parallel^2}{eB^2}\mathbf{J}_\perp$$

$$m\dot{v}_\parallel = -\hat{\mathbf{b}} \cdot (\mu \nabla B - e\mathbf{E})$$

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[c] A. Y. Aydemir, "A unified MC interpretation of particle simulations...", *Physics of Plasmas*, **1**, 1994

Slowing Down Distribution for Hot Particles

- for the slowing down distribution function

$$f_{eq} = \frac{P_0 \exp(\frac{P_\zeta}{\psi_0})}{\varepsilon^{3/2} + \varepsilon_0^{3/2}}$$

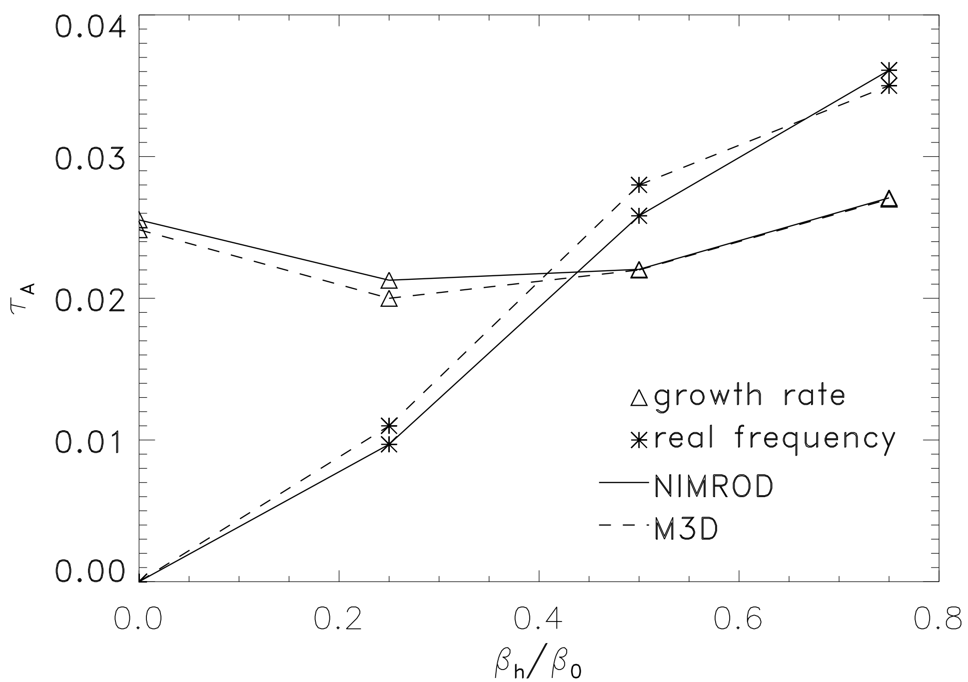
where $P_\zeta = g\rho_\| - \psi$ is the canonical toroidal momentum and ε is the energy, ψ_0 is the total flux, and ε_c is the critical slowing down energy

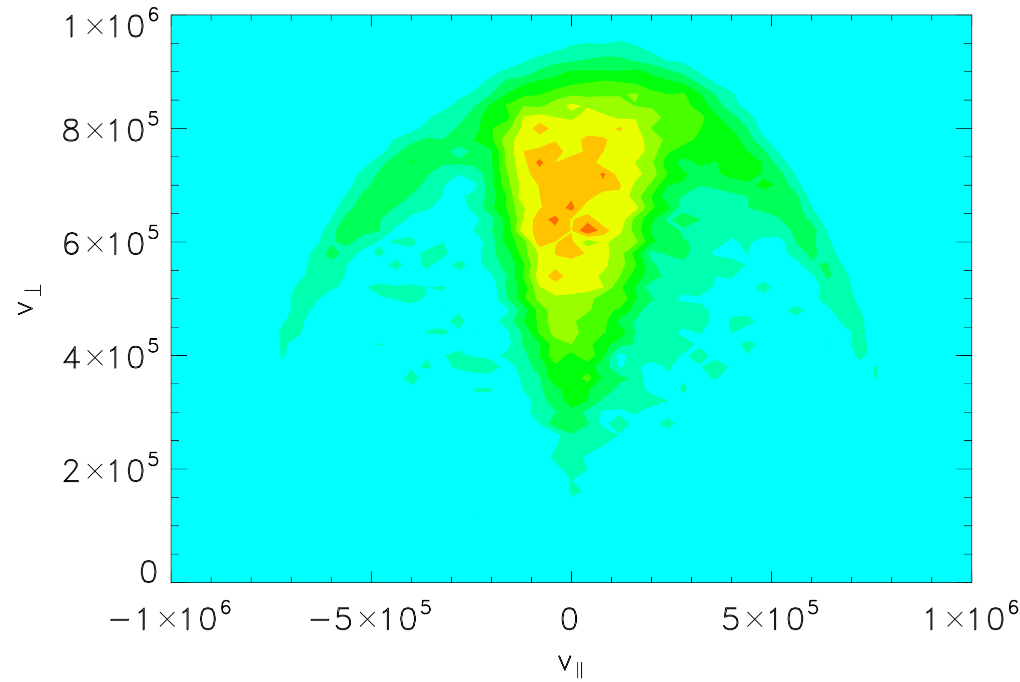
$$\begin{aligned} \delta\dot{f} &= f_{eq}\left\{\frac{mg}{e\psi_0 B^3}\left[\left(v_\|^2 + \frac{v_\perp^2}{2}\right)\delta\mathbf{B}\cdot\nabla B - \mu_0 v_\| \mathbf{J}\cdot\mathbf{E}\right]\right. \\ &\left. + \frac{\delta\mathbf{v}\cdot\nabla\psi_p}{\psi_0} + \frac{3}{2}\frac{e\varepsilon^{1/2}}{\varepsilon^{3/2}+\varepsilon_0^{3/2}}\mathbf{v}_D\cdot\mathbf{E}\right\} \end{aligned}$$

where

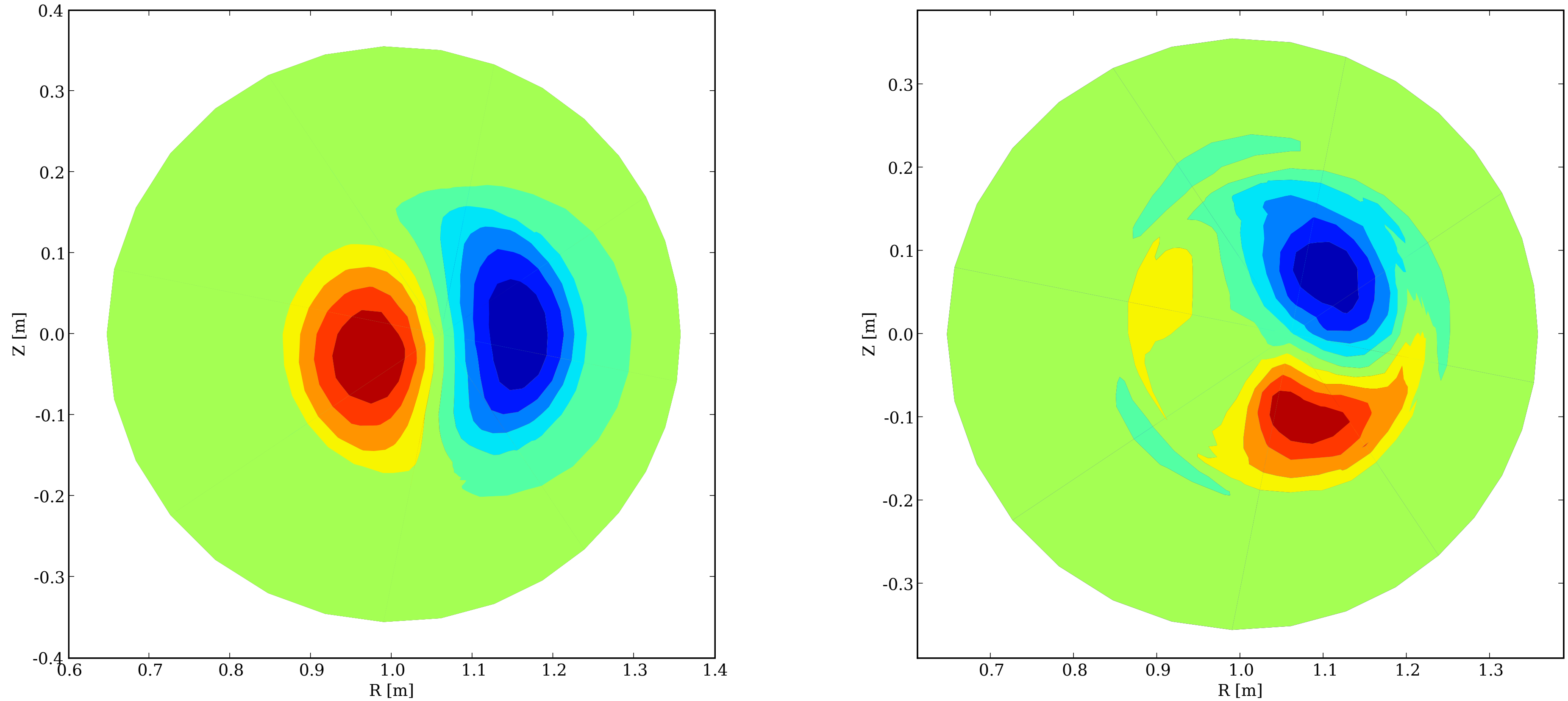
$$\begin{aligned} \mathbf{v}_D &= \frac{m}{eB^3}\left(v_\|^2 + \frac{v_\perp^2}{2}\right)(\mathbf{B}\times\nabla B) + \frac{\mu_0 m v_\|^2}{eB^2}\mathbf{J}_\perp \\ \delta\mathbf{v} &= \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \mathbf{v}_\| \cdot \frac{\delta\mathbf{B}}{B} \end{aligned}$$

Benchmark with M3D

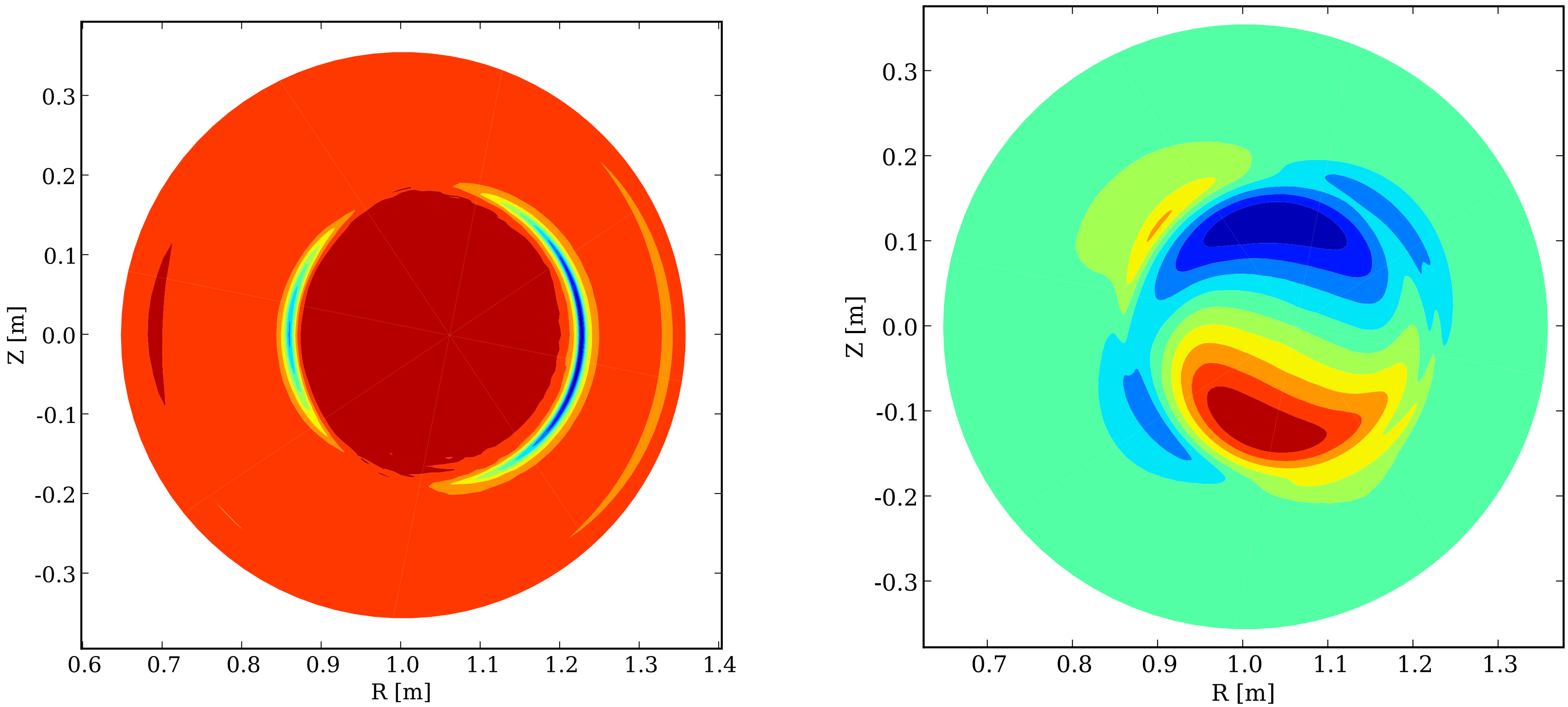




- improved particle loading - ensures velocity space isotropy
- velocity shows most activity in the most energetic particles
- mostly in trapped region, also in extremes of passing particles



- simulations without anisotropy reproduce ideal MHD with γ within 10%
- no real frequency
- anisotropic pressure $\Delta p = p_{\|} - p_{\perp}$ is key to energetic effects



- simulations with only passing particles stabilize but do not change V_ϕ mode topology
- trapped particles excite precessional fishbone mode (plot on right)

- for $\beta_{frac} > 25\%$ simulations with only passing particles entirely stable $\gamma \sim 0$
- with only trapped particles $v_{cutoff} = 1.e6$, $\gamma > \gamma_{ideal}$.
- with only trapped particles $v_{cutoff} = 1.5e6$, $\gamma < \gamma_{ideal}$.
- inclusion of more energetic trapped particles has stabilizing influence - in line with prevailing ideas of energetic particle effects on $(1,1)$
- at larger v_{cutoff} (stronger anisotropy) there exists a window of stability
- modest energy passing particles seem to stabilize $(1,1)$!

Linear Simulations of Tearing Modes in a RFP

- alpha model equilibrium $\nabla \times \mathbf{B} = \mu \mathbf{B} \quad \mu = 2\Theta \left[1 - \left(\frac{r}{a} \right)^{\alpha_0} \right]$
- parameters for straight cylinder

  $a = .5\text{m},\ B_0 = .3\text{T},\ \Theta = 1.75,\ \alpha_0 = 3,$

  $S = 1.e4,\ ka = 2,\ \gamma\tau_A = 1.3e-3$

- Boris push with orbit averaging to accommodates disparate time scales
- energetic ion density profile $\propto \exp\left[- \left(\frac{r}{0.45a} \right)^2 \right]$
- initialize with mono-energetic particles $\delta(\mathbf{v}_\perp - \mathbf{v}_0)$, only $\mathbf{v} \times \delta\mathbf{B}$ in weight equation
- use **only** perpendicular pressure for comparison with theory
- subcyling of particles and orbit average particle pressure

δf and the Lorentz Equations

- Lorentz equation of motion

$$\dot{\mathbf{x}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

- for Lorentz equations use[a]

$$f_{eq} = f_0(\mathbf{x}, v^2) + \frac{1}{\omega_c}\left(\mathbf{v} \cdot \mathbf{b} \times \nabla f_0\right)$$

- weight equation is

$$\dot{\delta f} = -\frac{\delta \mathbf{E} + \mathbf{v} \times \delta \mathbf{B}}{B} \cdot \mathbf{b} \times \nabla f_0 - \frac{2q}{m}\delta \mathbf{E} \cdot \mathbf{v}\frac{\partial f_0}{\partial v^2}$$

[a] M. N. Rosenbluth and N. Rostoker “Theoretical Structure of Plasma Equations”, Physics of Fluids **2** 23 (1959)

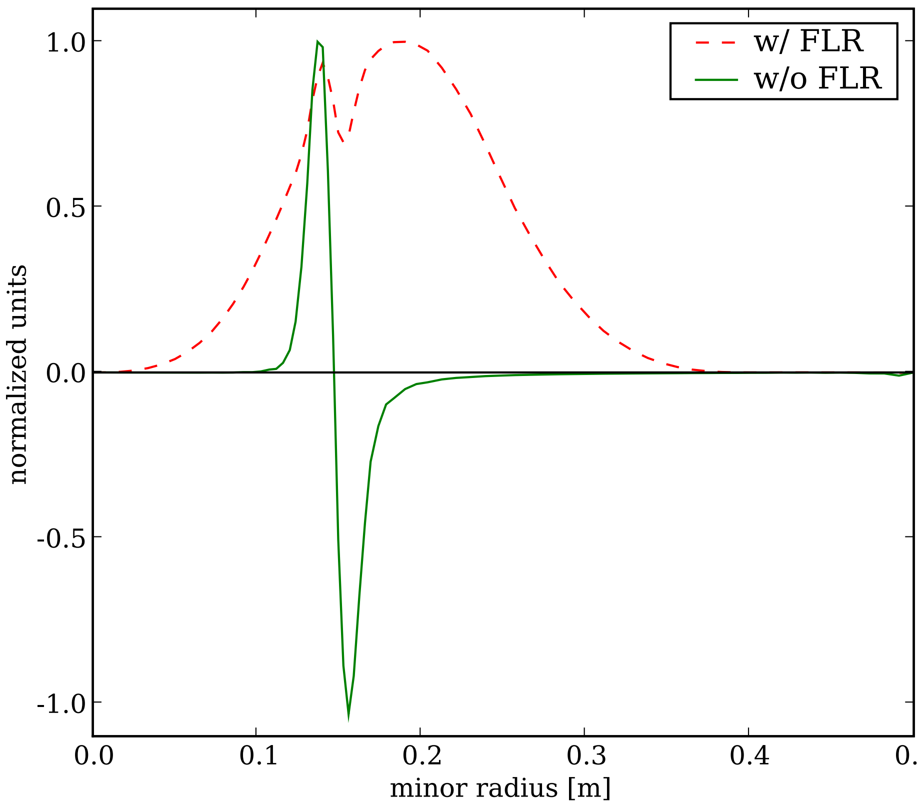
FLR Stabilization of RFP Tearing Mode

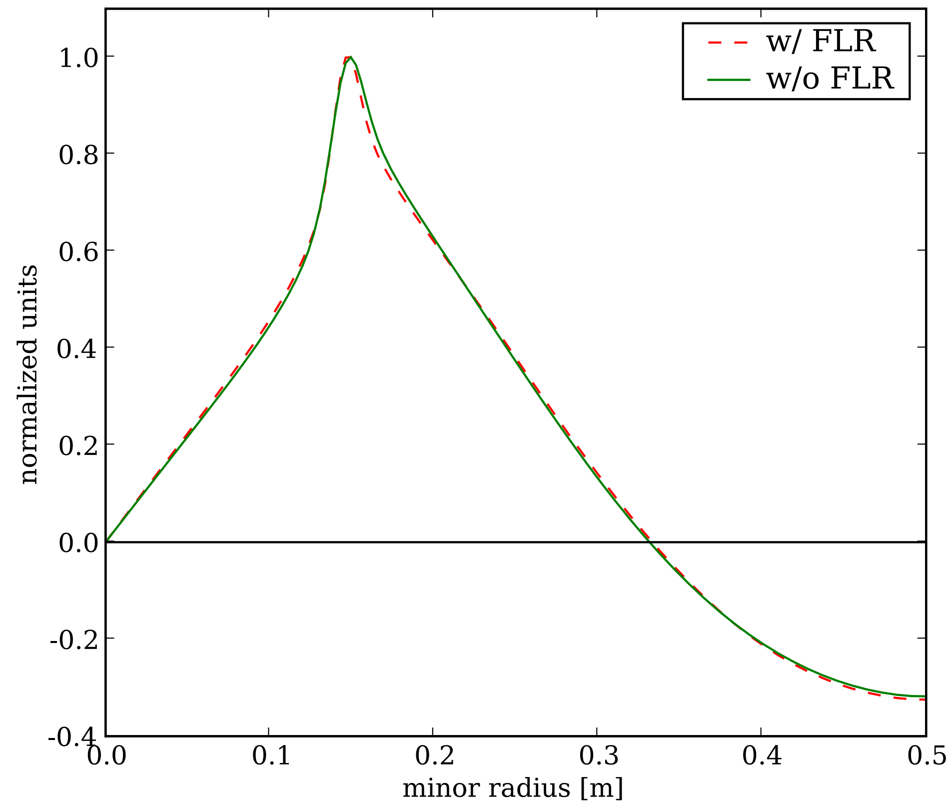
- stabilization with increasing $\mathbf{v}_\perp$

v_0(m/s)	L/a	$\gamma\tau_A$
base case	-	1.3×10^{-3}
1.0×10^6	.14	1.0×10^{-3}
1.5×10^6	.21	5.4×10^{-4}
2.0×10^6	.28	1.5×10^{-4}
2.5×10^6	.35	5.1×10^{-5}

- stabilization at $L/a \simeq 1/3$, where L is the Larmor diameter

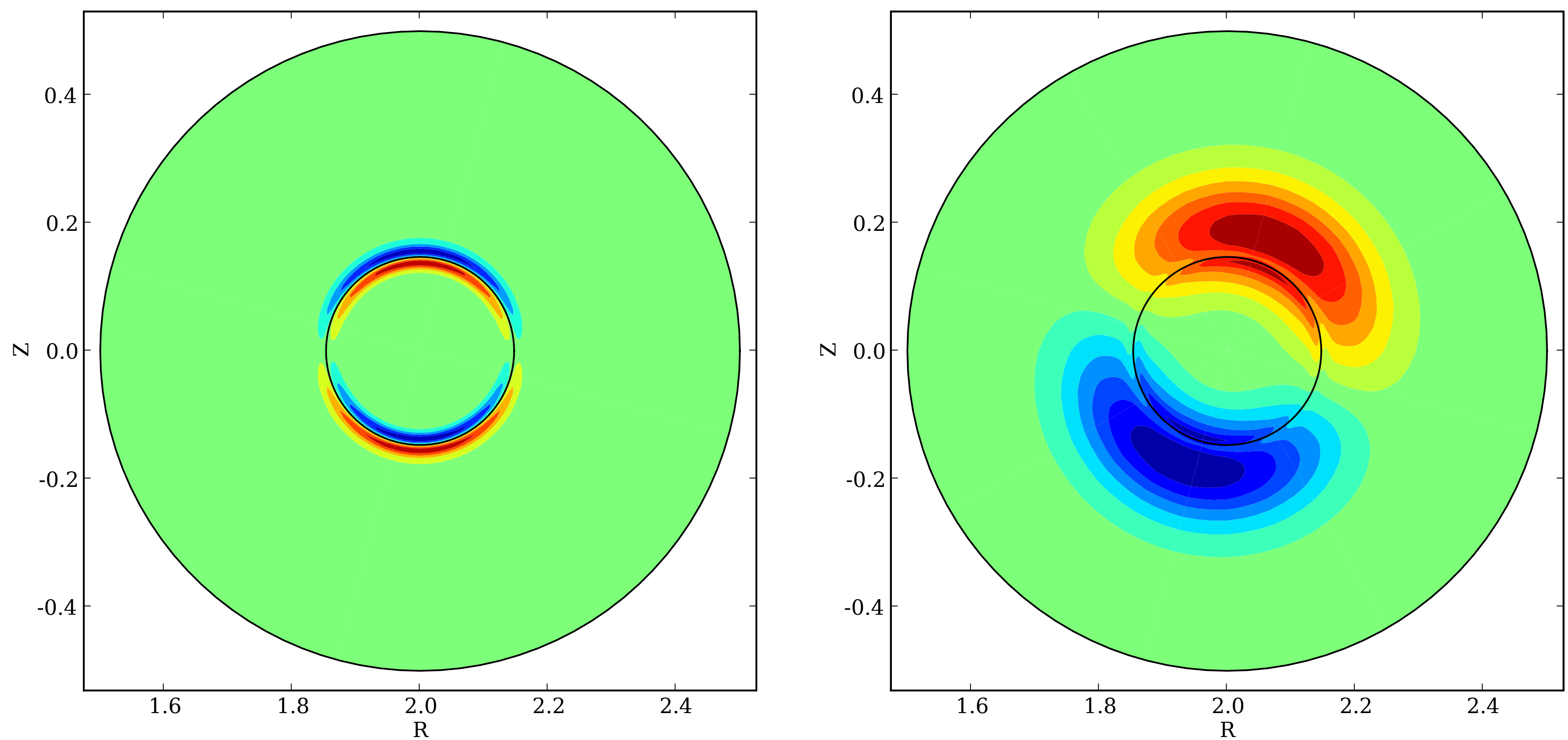
FLR Broadens Tangential Velocity Eigenmode Structure





- tangential velocity eigenmode substantially altered (left)
- magnetic eigenmode unaltered (right)

Comparison of V_ϕ Eigenmode



- inner circle shows resonance surface

The Future

- impact of localization of density
- full velocity distribution
- obtain MST equilibria and examine toroidicity - trapped particles
- visit in Sept-Oct