FRC Simulations with the NIMROD Code

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Abstract

The recently formed Plasma Science and Innovation Center (PSI-Center) is refining the NIMROD code to simulate Field Reversed Configurations (FRCs). The NIMROD code[1] can resolve highly anisotropic heat conduction and viscosity. This combined with its ability to include two-fluid effects allows us to capture more detailed physics than previous calculations. Some initial simulations are focused on 2D (n=0 only) nonlinear two-fluid simulations. In these initial calculations we focus on FRC formation in a theta-pinch, and the physics of spin-up due to end-shorting. Long-term plans include simulating the physics of toroidal field generation during FRC formation – especially when a non symmetric formation technique (such as a conical theta-pinch) is employed to form a translating FRC. We will also try to simulate the generation of poloidal flow, which together with toroidal field, might be essential in capturing FRC stability[2].

At the same time, a new time-implicit two-fluid version of NIMROD is being verified and applied to the macroscopic stability of a FRC. FRC macro-stability is a problem of intrinsic interest, both from a fundamental and from a practical standpoint, and also presents a unique verification opportunity. Initial $n = 1$ linear and nonlinear results have reproduced earlier observed[3,4] transition from the “fundamental” MHD internal tilt mode to modes with higher structure along B, with growth rates smaller but comparable to MHD growth rates. Previous long-thin analysis[5] has been extended to capture the features of these modes and results are compared with NIMROD. Modifications to the analysis suggest effects beyond HMHD, which are important for the stability of these modes. Such effects will be incorporated into future NIMROD versions and verified by comparison with the analysis.

[1] C.R. Sovinec et al. J. Comp. Phys. **195**, 355 (2004)

[2] L. C. Steinhauer, Phys. Plasmas **6**, 2734 (1999)

[3] R.D. Milroy, D.C. Barnes, R.C. Bishop, and R.B. Webster, Phys. Fluids **B1**, 1225 (1989)

[4] Elena V. Belova et al. Phys. Plasmas **10**, 2361 (2003)

[5] D. C. Barnes, Phys. Plasmas 10, 1636 (2003)

Background: The FRC as a Tool for Code Development

- FRCs provide stringent verification of the reliability of FLR and kinetic effects in numerical simulations
- Highly kinetic device with large Larmor radii where the Hall term becomes potent
- High Beta
- Large density gradients
- Simply connected

NIMROD

- Extended MHD code that includes: Hall physics, gyroviscosity, anisotropic heat conduction, temperature dependent resistivity, non-local and particle closures for higher order kinetic effects.
- High order finite elements in two dimensions and spectral in the third dimension allows for 2D and 3D functionality.
- Semi-implicit leap frog time stepping is used for stability beyond the CFL threshold.
- Operates both linearly and nonlinearly.
- Modular Fortran 90 implementation.

*C.R. Sovinec et al. J. Comp. Phys. **195**, 355 (2004)

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NIMROD Equations

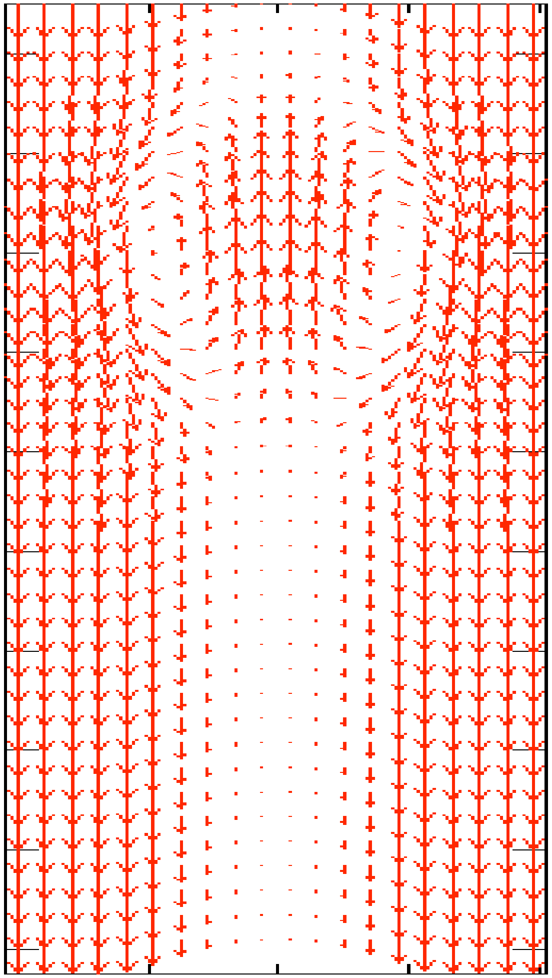
- Continuity: $\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{u}) = 0$
- Momentum: $\rho\left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u}\right) = \vec{J} \times \vec{B} - \nabla P - \nabla \cdot \overleftrightarrow{\Pi}$
- Temperature: $\frac{n_s}{\gamma - 1}\left(\frac{\partial}{\partial t} + \vec{u} \cdot \nabla\right) T_s = -P_s \nabla \cdot \vec{u}_s - \overleftrightarrow{\Pi}_s : \nabla \cdot \vec{u}_s - \nabla \vec{q}_s + Q_s$
- Faraday's Law: $\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$
- Ohms Law:

$$\vec{E} = -\vec{V} \times \vec{B} + \eta \vec{J} + \frac{1}{ne}\left(\vec{J} \times \vec{B} - \nabla P_e\right) + \frac{m_e}{ne^2}\left(\frac{\partial \vec{J}}{\partial t} + \nabla \cdot \left(\overleftrightarrow{J}\vec{V} + \overleftrightarrow{V}\vec{J}\right)\right) + \nabla \overleftrightarrow{\Pi}$$

*C.R. Sovinec et al. J. Comp. Phys. **195**, 355 (2004)

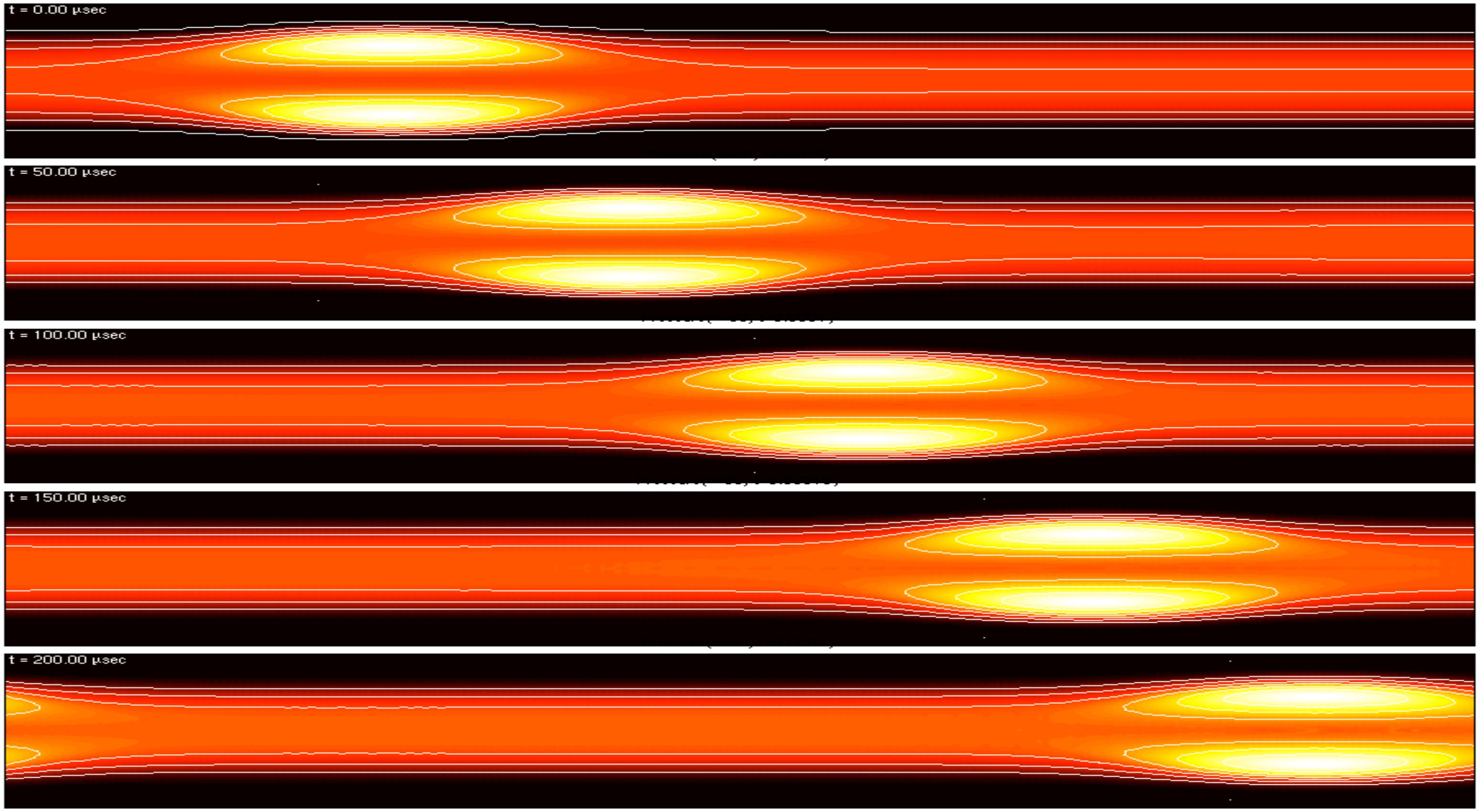
FRC Translation

- The MTF, PHD, and TCSU devices contain translating FRCs
- Conservation of mass and magnetic flux is an important validation of a code's ability to treat translation.
- n=0, non-linear MHD with anisotropic heat conduction.
- The FRC is given an initial velocity around a stationary background.



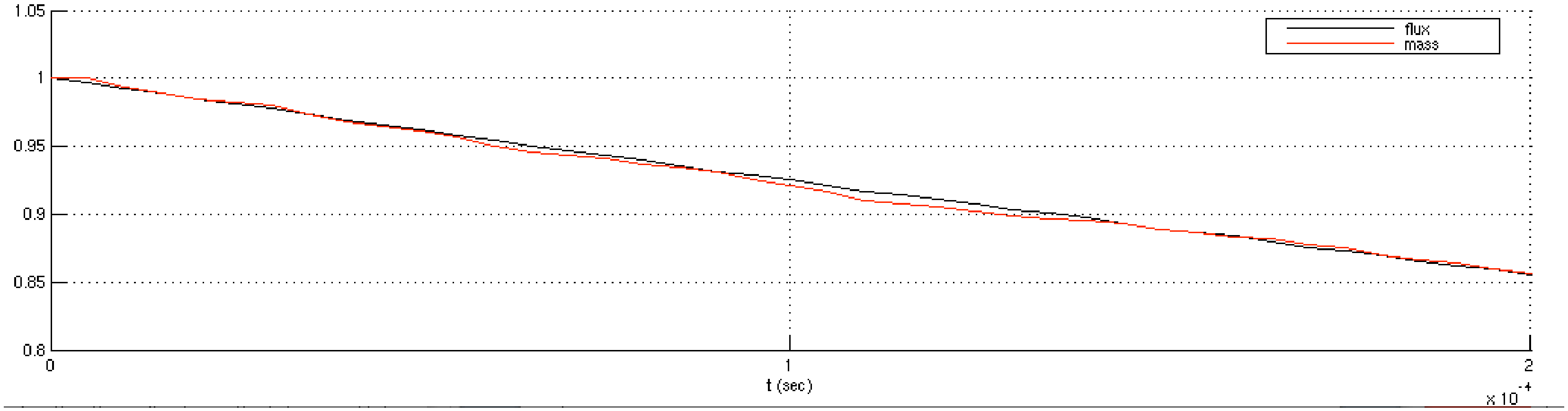
Pressure Contours Translating

- T_e=T_i=100 eV, Density = 1.24e20m^{-3}, B=0.1 T, r_s=0.65m.



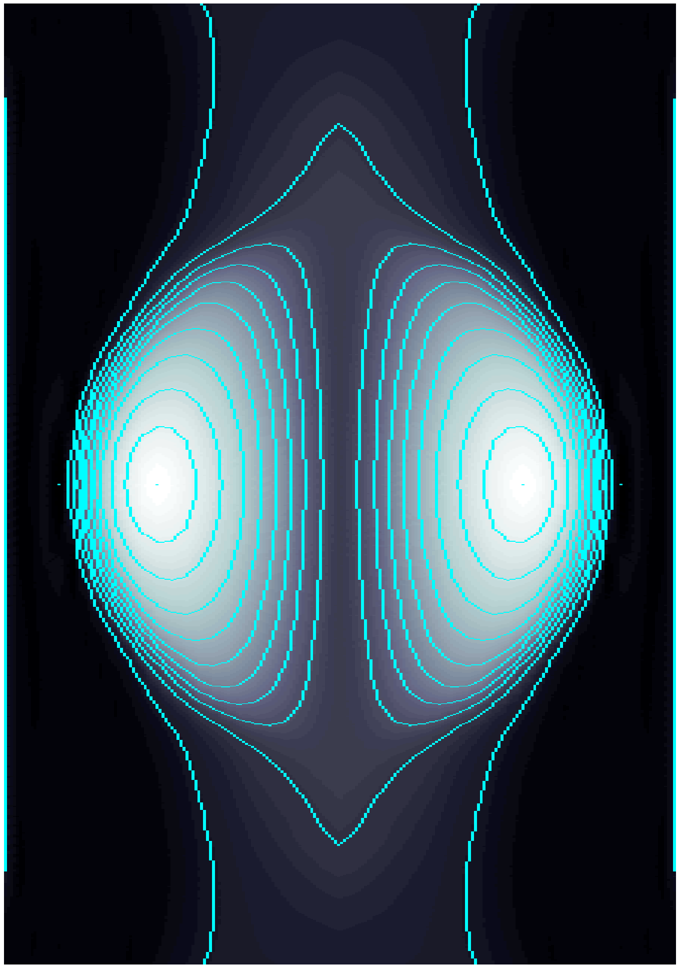
Conservation of Density and Magnetic Flux

- For $\frac{\eta}{\mu} = 3$, simple calculations indicate that 85% of the flux should be retained after 2.0E-4 s.
- Mass loss can reasonably be assumed to match the flux loss inside the FRC.
- Very little magnetic flux or mass loss due to convection



Anisotropic Thermal Conduction

- The NIMROD code is able to close the pressure evolution equation with anisotropic thermal transport.
- Heat is conducted much more quickly along the magnetic field lines.
- Produces a more realistic fluid description of highly kinetic plasmas



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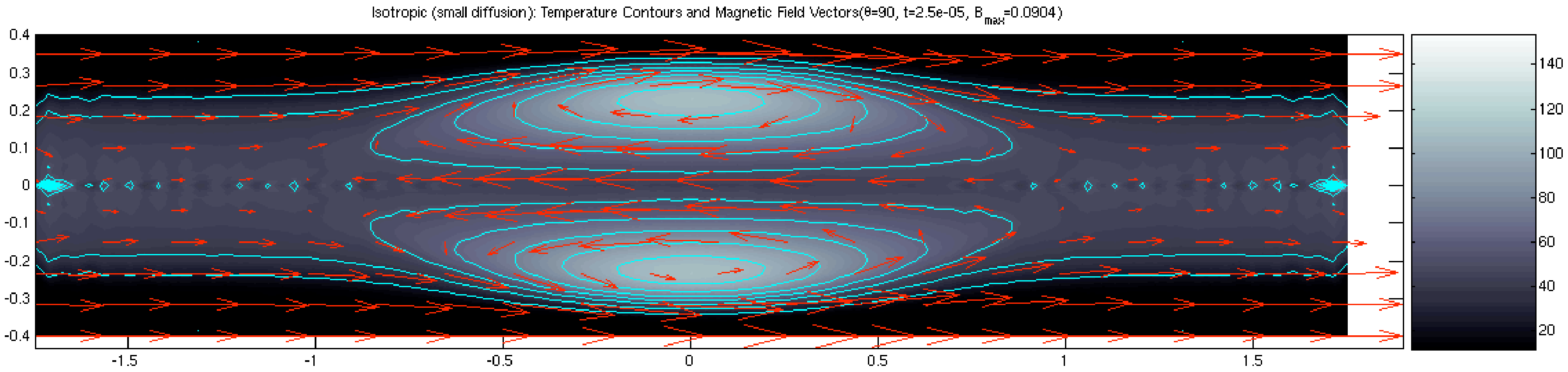
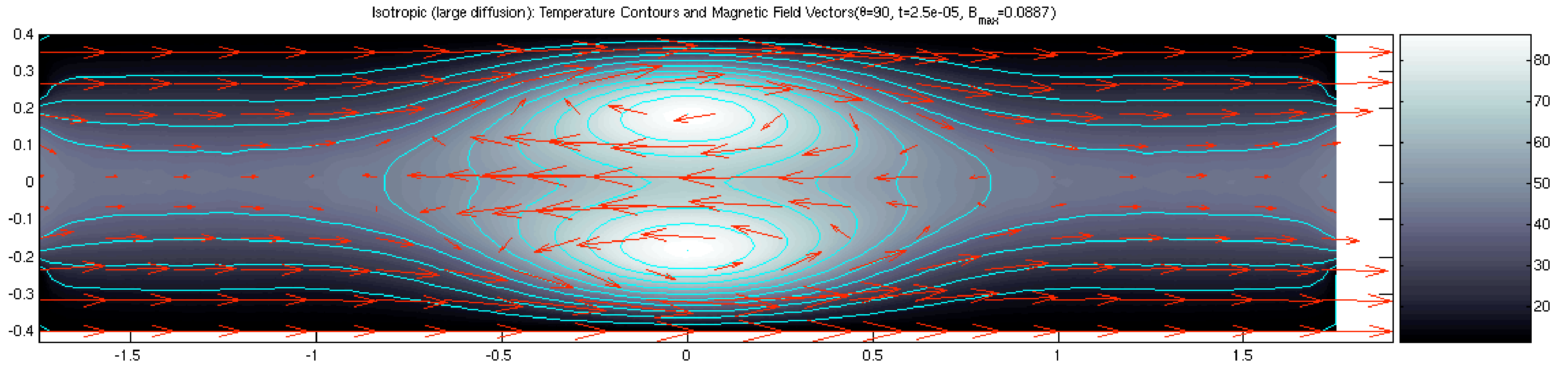
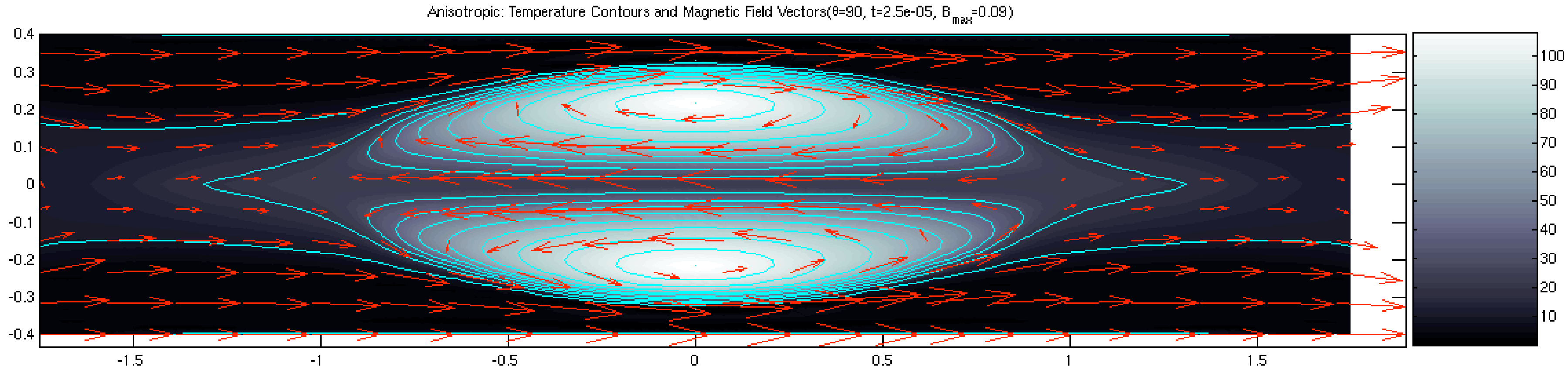
Comparison of Thermal Conduction

- With anisotropic heat diffusion the contours of temperature have diffused along.
- The magnetic field vector plots match the temperature contours very closely
- Figure 1: isotropic k=2, Figure 2: isotropic k=200, Figure 3: anisotropic k_{perp}=2, $k_{parallel}$=1e5 in m^2/s.


Isotropic (small diffusion): Temperature Contours and Magnetic Field Vectors(θ=90, t=2.5e-05, B_{max}=0.0904)


Isotropic (large diffusion): Temperature Contours and Magnetic Field Vectors(θ=90, t=2.5e-05, B_{max}=0.0887)


Anisotropic: Temperature Contours and Magnetic Field Vectors(θ=90, t=2.5e-05, B_{max}=0.09)

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End Shorting and FRC Spin-Up

- Toroidal spin-up in the FRC has been attributed both to particle loss and to end shorting
- The constant E_r end boundary condition couples to produce a B_{theta} and JxB, leading to toroidal spin-up in the open field line region.
- The open field line region couples to the FRC through the fluid viscosity.
- Here we compare spin-up due only to end shorting against a case with periodic boundary conditions.
- No rotation is generated if the Hall term is not included.

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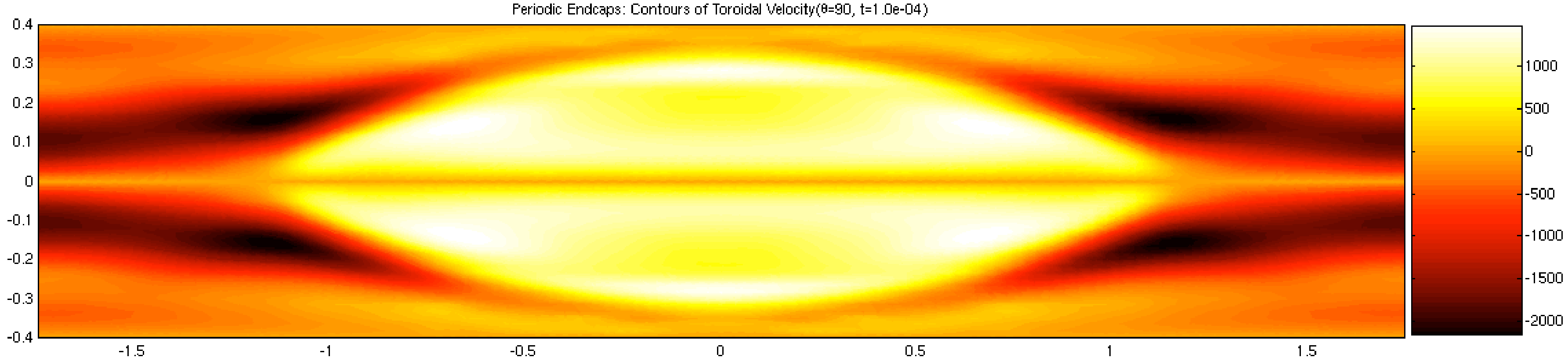
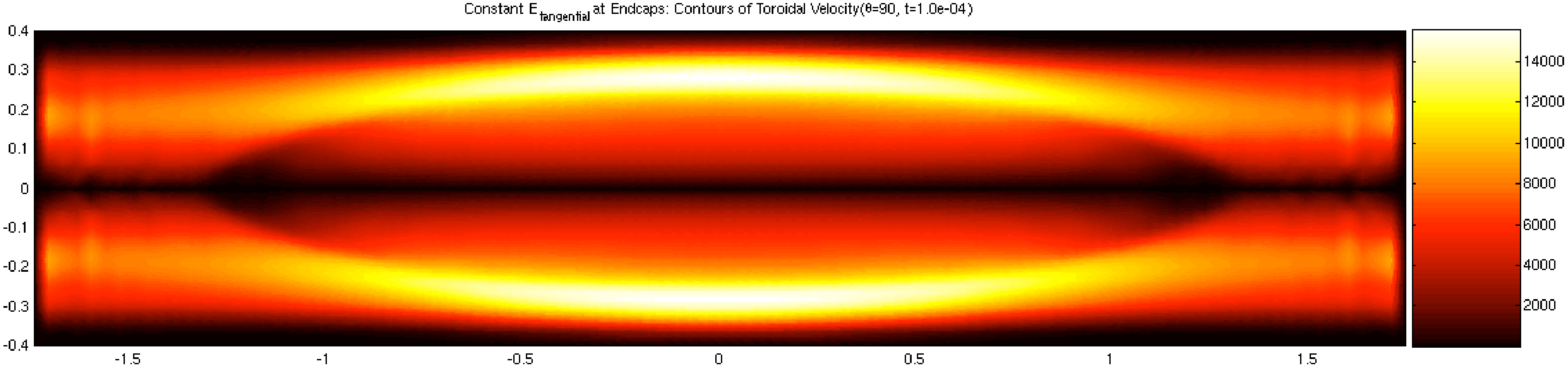
Spin-Up: No End Shorting vs. End Shorting



Periodic Endcaps: Contours of Toroidal Velocity(θ=90, t=1.0e-04)



Constant $E_{tangential}$ at Endcaps: Contours of Toroidal Velocity(θ=90, t=1.0e-04)

$T_e=T_i=100$ eV, Density = 1.24e20m^{-3}, B=0.1 T, $r_s=0.8$m

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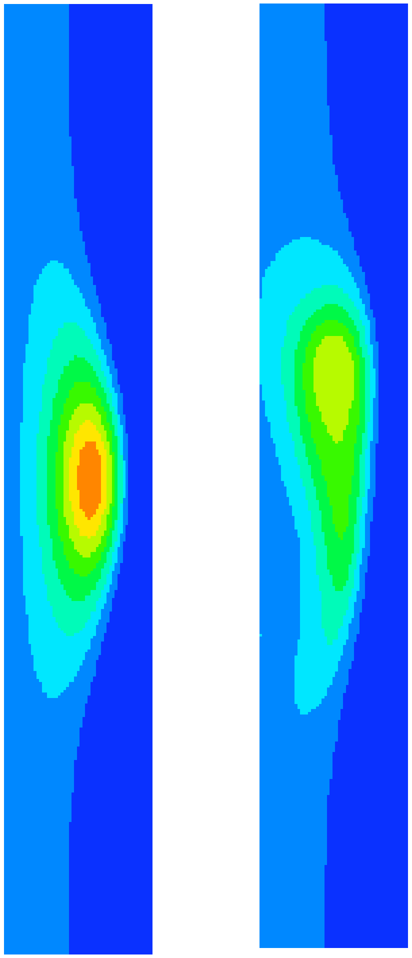
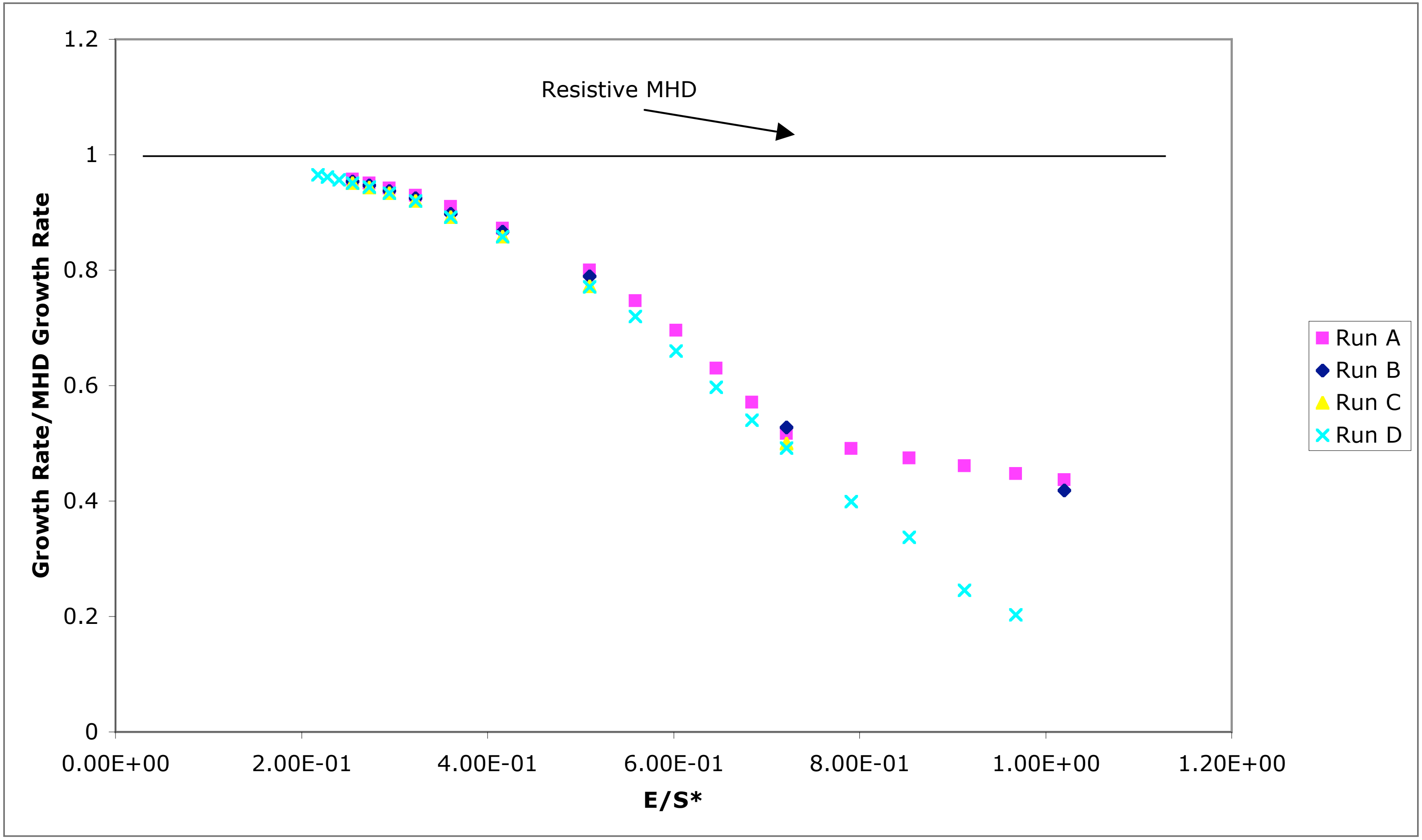
The FRC Tilt Mode

- Most dangerous global mode for FRCs.
- n=1
- Is not easily verified by experiment.
- Actual growth times are longer than predictions of resistive MHD.
- Two-fluid, FLR and kinetic effects slow the mode's growth.



Effect of Hall Physics on Growth



Hall MHD Model of Tilt Mode Growth

- Extension of MHD to include $\vec{J}x\vec{B}$ and neglect ∇P gives the dispersion relation:

$$\omega^2 + \frac{a\gamma_{MHD}}{S^* / E}\omega + \gamma_{MHD}^2$$

- Solving for the growth rate gives:

$$\gamma^2 = \gamma_{MHD}^2 \left[1 - \left(\frac{a}{S^* / E} \right) \right]$$

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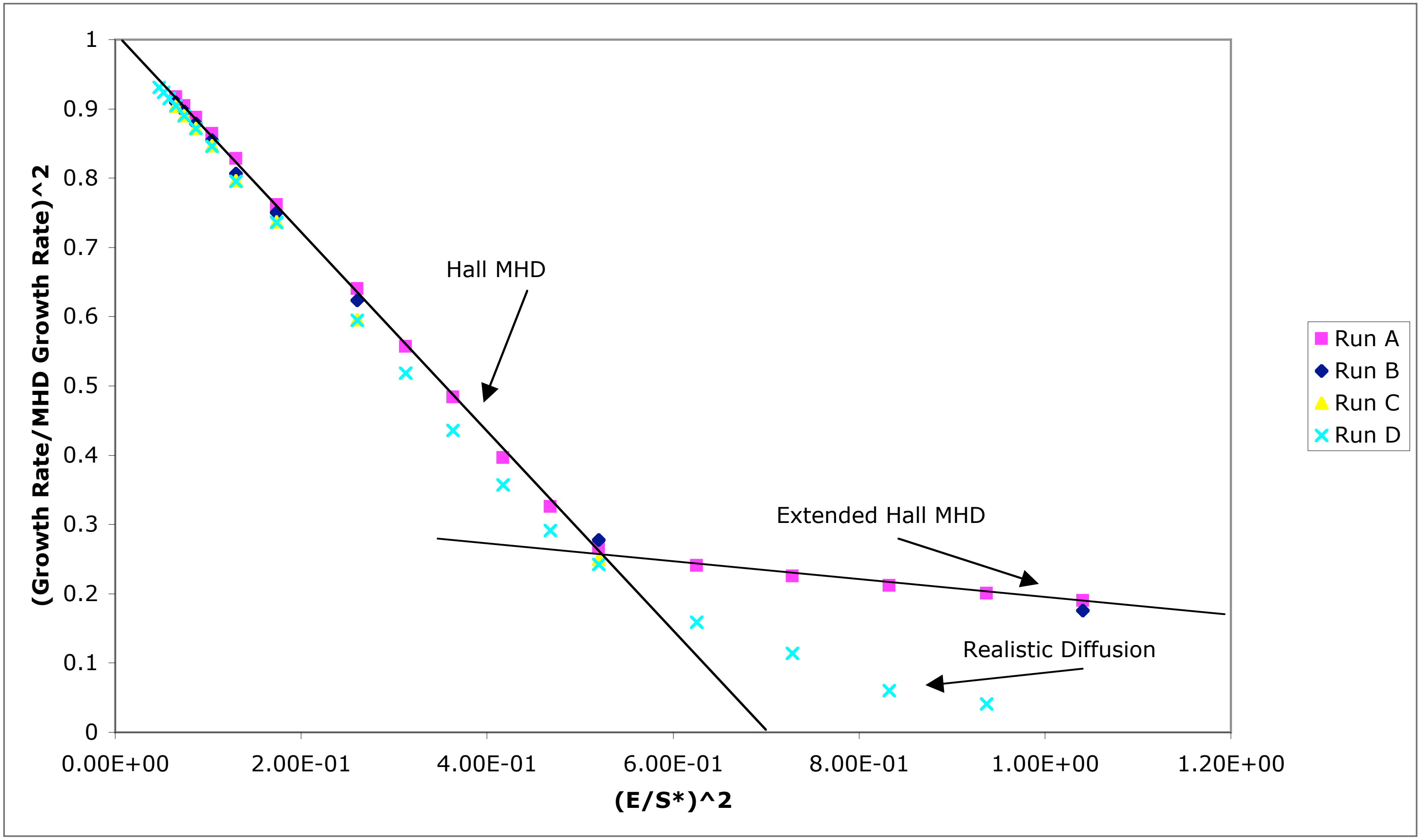
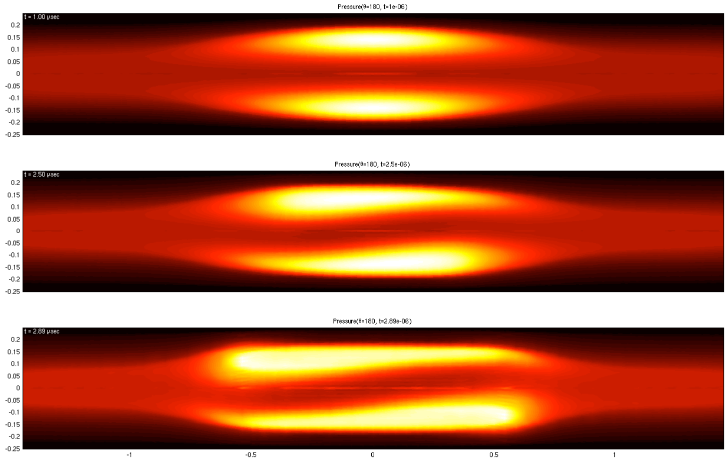
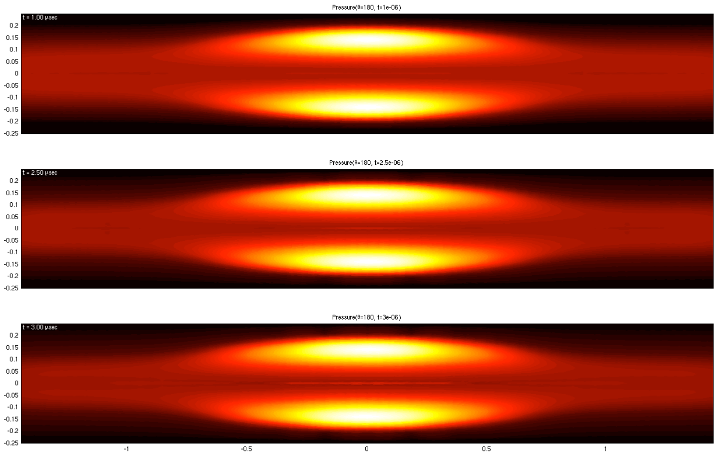
Hall Theory for Large E/S*



MHD and Hall MHD Pressure Profiles

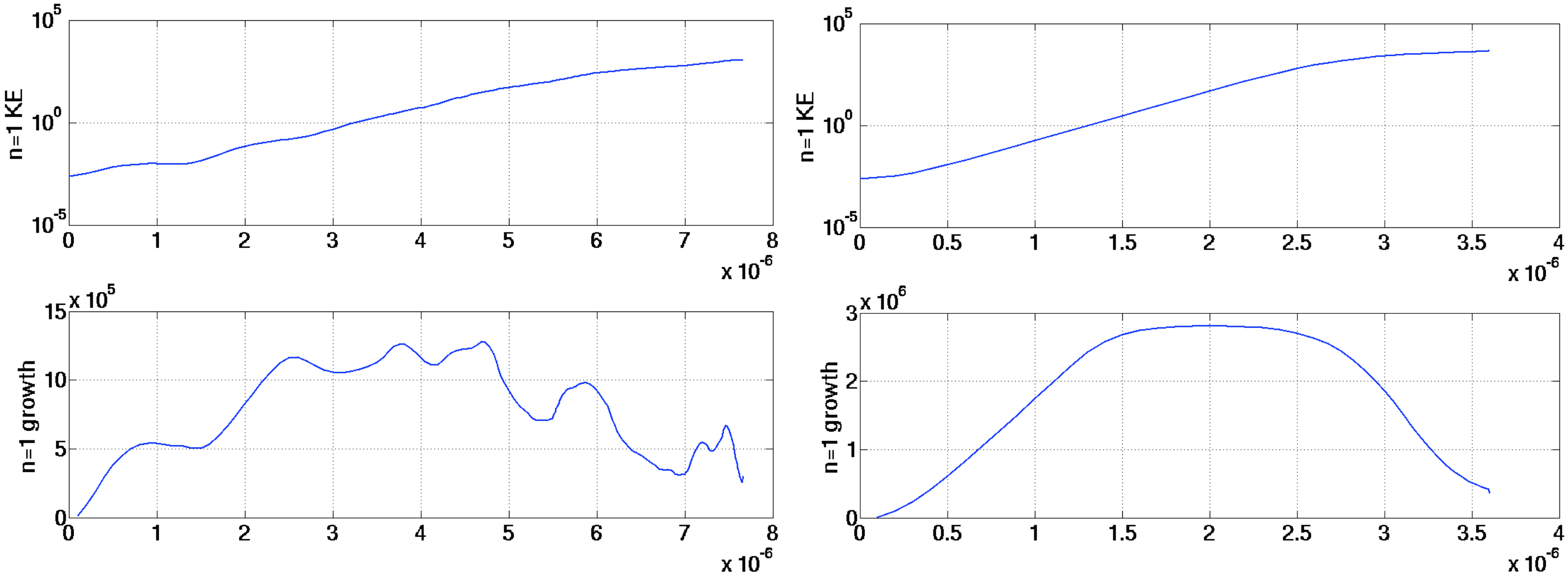
- The growth of the tilt mode in non-linear simulations is clearly suppressed with Hall MHD (right) as compared against MHD (left).
- Nonlinear 3-D Hall physics significantly retards, but does not eliminate the growth of the FRC tilt mode.
- viscosity=100 m^2/s, electric diffusivity= 10 m^2/s, density=6.25e19 m^{-3}





MHD and Hall MHD n=1 Growth

- The growth of the tilt mode in non-linear simulations is clearly suppressed with Hall MHD (right) as compared against MHD (left)



Summary

- Significant progress made in the adding Hall physics to NIMROD (Barnes & Sovinec).
- Anisotropic thermal conduction readily smoothes temperature profiles along FRC field lines.
- NIMROD can translate an FRC with good mass and magnetic flux conservation.
- Hall MHD simulations show that significant spin-up is achieved via an end shorting mechanism.
- Hall MHD simulations show a slowing in the growth rate of the FRC tilt mode.

Future work

- Initiate FRC translation with a radial boundary condition
- Examine spin-up via end shorting in translating and RMF FRCs
- Develop a model to explain the growth rate of the tilt mode in the high E*/S regime.
- Examine the degree to which ion gyroviscosity stabalizes the FRC tilt mode.
- Continue to develop, validate and verify kinetic physics for ICC devices.

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