Hybrid Kinetic-MHD Simulations at the PSI Center

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NIMROD
Non-Ideal MHD with Rotation
Open Discussion Project

Plasma Science and Innovation
PSI-Center

The Outline

- summary of theory FLR effects on tearing modes (V. Svidzinski, PoP 2004)
- NIMROD preliminaries
- hybrid kinetic-MHD
- initial simulation results
- stepping backwards to a slab
- stepping forward - plans

Summary of Theoretic Prediction of V. Svidzinski[a]

- predicts that in the limit of $v_{\|} = 0$, large FLR orbits stabilize tearing modes
- treats the energetic particles as a perturbation that modifies tearing layer problem in RFP
  - backs out a conductivity tensor from linearized Vlasov equation
  - uses conductivity tensor to obtain $\mathbf{J}_{hot}$
  - uses Ampere's law to obtain energetic particle corrections to magnetic field and growth rate in linearized MHD equations
- examines impact of localized energetic distribution
- conjectures that finite $v_{\|}$ dilutes stabilization

[a]V. A. Svidzinski and S. C. Prager, "Effects of particles with large gyroradii on resistive magnetohydrodynamic stability", PoP **11** 980, 2004

NIMROD Preliminaries

- NIMROD is an initial value 3D XMHD code
- uses finite elements in two dimension, Fourier in the third
  - allows geometric flexibility
  - can handle extreme anisotropies, $\frac{\chi_\parallel}{\chi_\perp} \gg 10^6$
- semi-implicit advance, not restricted by magnetosonic CFL condition
  - model experiment relevant parameters, $S \sim 10^{7-8}$
- allows both linear and nonlinear simulations

NIMROD

- NIMROD's extended MHD equations

$$
\begin{aligned}
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla \times \mathbf{E} + \kappa_{divb} \nabla\nabla \cdot \mathbf{B} \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} \\
\mathbf{E} &= -\mathbf{U} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} \\
+\frac{m_e}{ne^2}\left[\sum_\alpha \frac{q_\alpha}{m_\alpha}\left(\nabla p_\alpha + \nabla \cdot \Pi_\alpha\right)\right] &+ \frac{m_e}{ne^2}\left[\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot \left(\mathbf{JU} + \mathbf{UJ}\right)\right] \\
\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{U}) &= \nabla \cdot D \nabla n \\
mn\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U}\right) &= \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \rho\nu\nabla\mathbf{V} - \nabla \cdot \Pi - \nabla \cdot p_h \\
\frac{n_\alpha}{\Gamma - 1}\left(\frac{\partial T_\alpha}{\partial t} + \mathbf{U}_\alpha \cdot \nabla T_\alpha\right) &= -p_\alpha \nabla \cdot \mathbf{U}_\alpha - \nabla \cdot q_\alpha + Q_\alpha - \Pi_\alpha : \nabla \mathbf{U}_\alpha
\end{aligned}
$$

Representation of NIMROD fields

- NIMROD fields use quadrilateral finite element-Fourier representation

$$\delta A(\mathbf{x},t) = \sum_j A_{j,0}(t)\alpha_{j,0} + \sum_j \sum_n (A_{j,n}(t)\alpha_{j,n} + c.c.)$$

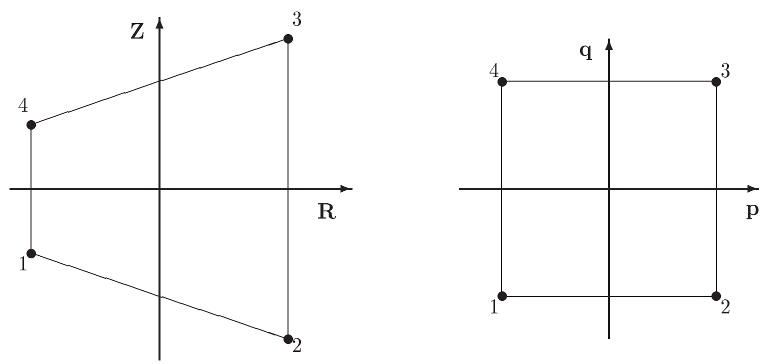
where

$$\alpha_{j,n} = N_j(p,q)\exp(in\phi)$$

(p,q) are logical coordinates, $N_j(p,q) = l_j(p)l_j(q)$ and

$$l_i(x) = \prod_{i=0,i\neq j}^{k} \frac{x - x_i}{x_j - x_i}$$

$k = pd + 1$, pd is the polynomial order of the Lagrange polynomial



Summary of the δf PIC Method[a b c]

- PIC is a Lagrangian simulation of phase space $f(\mathbf{x},\mathbf{v}) \rightarrow f(\mathbf{x}(t),\mathbf{v}(t))$
  - discretize $f(\mathbf{x},\mathbf{v})$ - sample with particles
  - equations of motion are used to advance particles
  - PIC uses spatial grid for field information
- in principle, $f(\mathbf{x},\mathbf{v})$ contains everything
- **BUT** limited statistics - PIC is noisy, can't beat $1/\sqrt{N}$
- δf PIC reduces the discrete particle noise associated with conventional PIC

[a] S. E. Parker and W. W. Lee, 'A fully nonlinear characteristic method for gyro-kinetic simulation', *Physics of Fluids B*, **5**, 1993

[b] G. Hu and J. A. Krommes, "Generalized weighting scheme for δf particle simulation method", *Physics of Plasmas*, **1**, 1994

[c] A. Y. Aydemir, "A unified MC interpretation of particle simulations...", *Physics of Plasmas*, **1**, 1994

The δf PIC Method cont.

- begin with the Vlasov Equation

$$\frac{\partial f(\mathbf{z})}{\partial t} + \dot{\mathbf{z}} \cdot \frac{\partial f(\mathbf{z})}{\partial \mathbf{z}} = 0$$

- split distribution function into steady state and evolving perturbation:
  - $f = f_{eq}(\mathbf{z}) + \delta f(\mathbf{z}, t)$
  - moments of f_{eq} easy to compute

- δf evolves along the characteristics $\dot{\mathbf{z}}$ i.e. the equations of motion

$$\dot{\delta f} = -\dot{\tilde{\mathbf{z}}} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}}$$

where $\dot{\mathbf{z}} = \dot{\mathbf{z}}_{eq} + \dot{\tilde{\mathbf{z}}}$

- must choose a stationary f_{eq} ($\dot{\mathbf{z}}_{eq} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}} = 0$), often chosen as a function of canonical variables

δf and the Lorentz Equations

- Lorentz equation of motion

$$\dot{\mathbf{x}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

- for full kinetic equations use[a]

$$f_0 = f(\mathbf{x}, v^2) + \frac{1}{\omega_c}\left(\mathbf{v} \cdot \mathbf{b} \times \nabla f\right)$$

- weight equation is

$$\dot{\delta f} = -\frac{\delta \mathbf{E} + \mathbf{v} \times \delta \mathbf{B}}{B} \cdot \mathbf{b} \times \nabla f - \frac{2q}{m} \delta \mathbf{E} \cdot \mathbf{v} \frac{\partial f}{\partial v^2}$$

- reproduces drift kinetic results for energetic particle effect on $(1,1)$ internal kink mode

- compute perturbed energetic pressure pressure moment of δf particles

[a] M. N. Rosenbluth and N. Rostoker "Theoretical Structure of Plasma Equations", Physics of Fluids **2** 23 (1959)

Simulation Details

- Boris push with orbit averaging to accomodates disparate time scales
- energetic ion density profile $\propto \exp\left[-\left(\frac{r}{0.45a}\right)^2\right]$
- use hybrid kinetic-MHD equations[a]

$$\rho\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U}\right) = \mathbf{J}\times\mathbf{B} - \nabla\cdot\underline{\mathbf{p}}_b - \nabla\cdot\underline{\mathbf{p}}_h$$

- project velocities into $(\perp, \|)$ (wrt $\mathbf{B}$)components
- use CGL pressure tensor $\delta\underline{\mathbf{p}}_h = \begin{pmatrix} p_\perp & 0 & 0 \\ 0 & p_\perp & 0 \\ 0 & 0 & p_\| \end{pmatrix}$
- ultimately want full pressure tensor (work in progress)

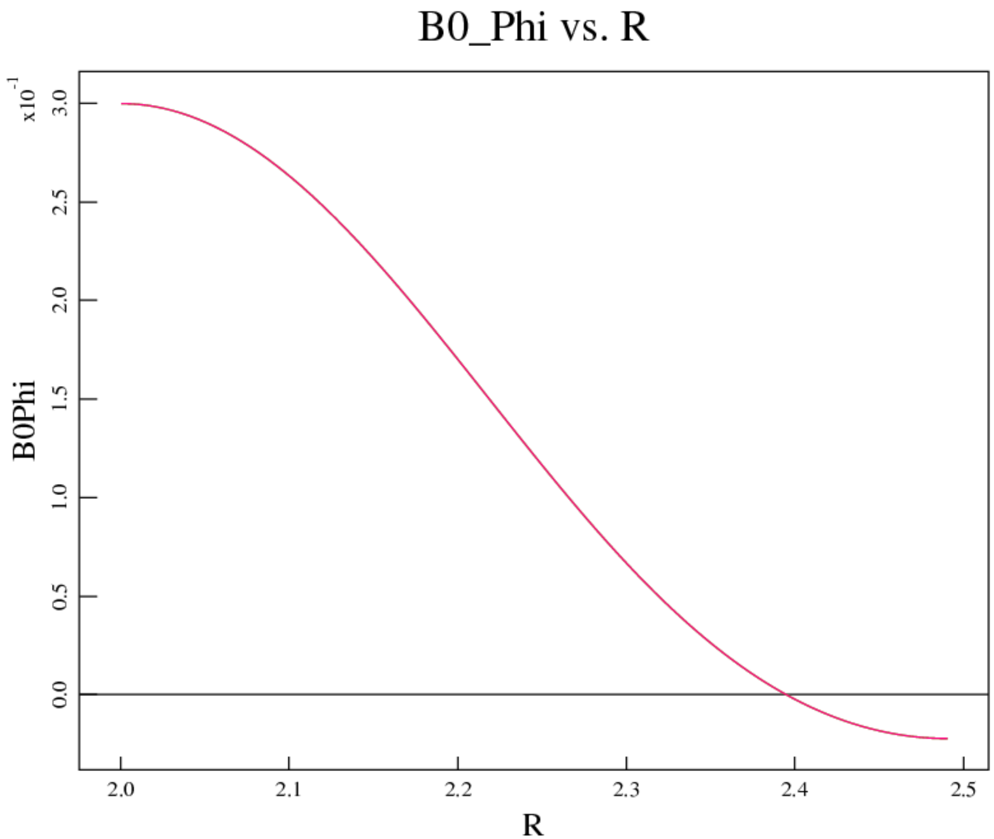
[a] C.Z.Cheng,"A Kinetic MHD Model for Low Frequency Phenomena",*J. Geophys. Rev*, **96**, 1991

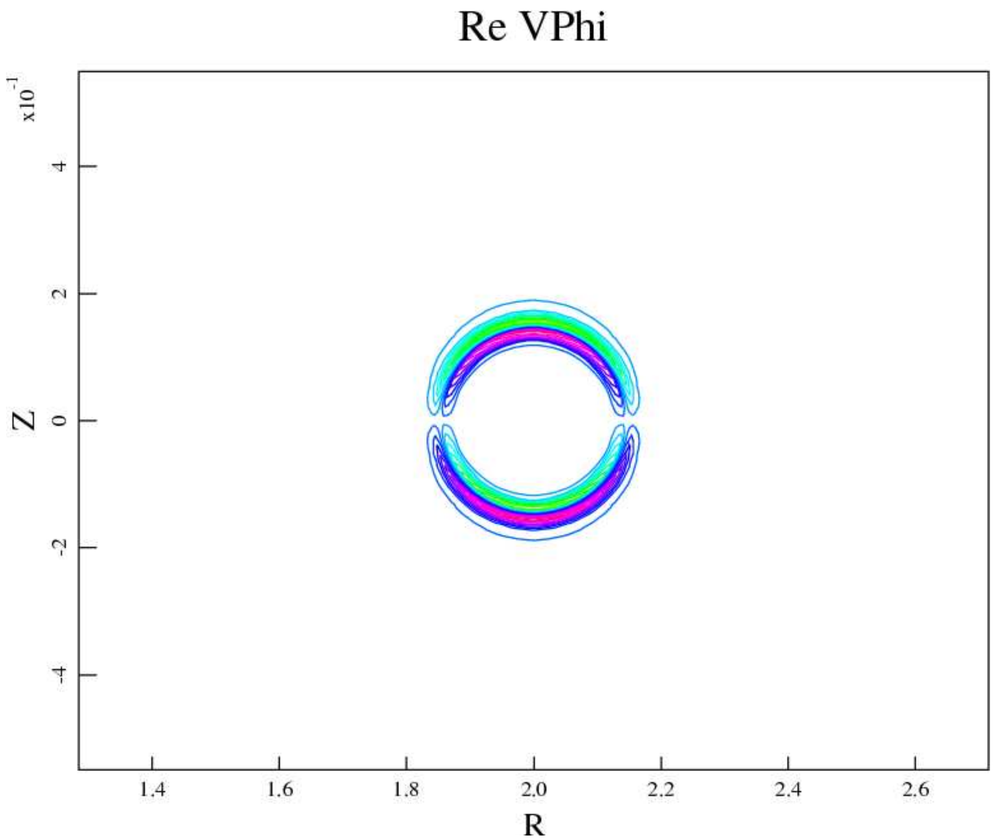
Linear Simulations of Tearing Modes in a RFP

- alpha model equilibrium $\nabla \times \mathbf{B} = \mu \mathbf{B} \quad \mu = 2\Theta \left[1 - \left(\frac{r}{a} \right)^{\alpha_0} \right]$
- parameters for straight cylinder

$$a = .5m,\ B_0 = .3T,\ \Theta = 1.75,\ \alpha_0 = 3,$$

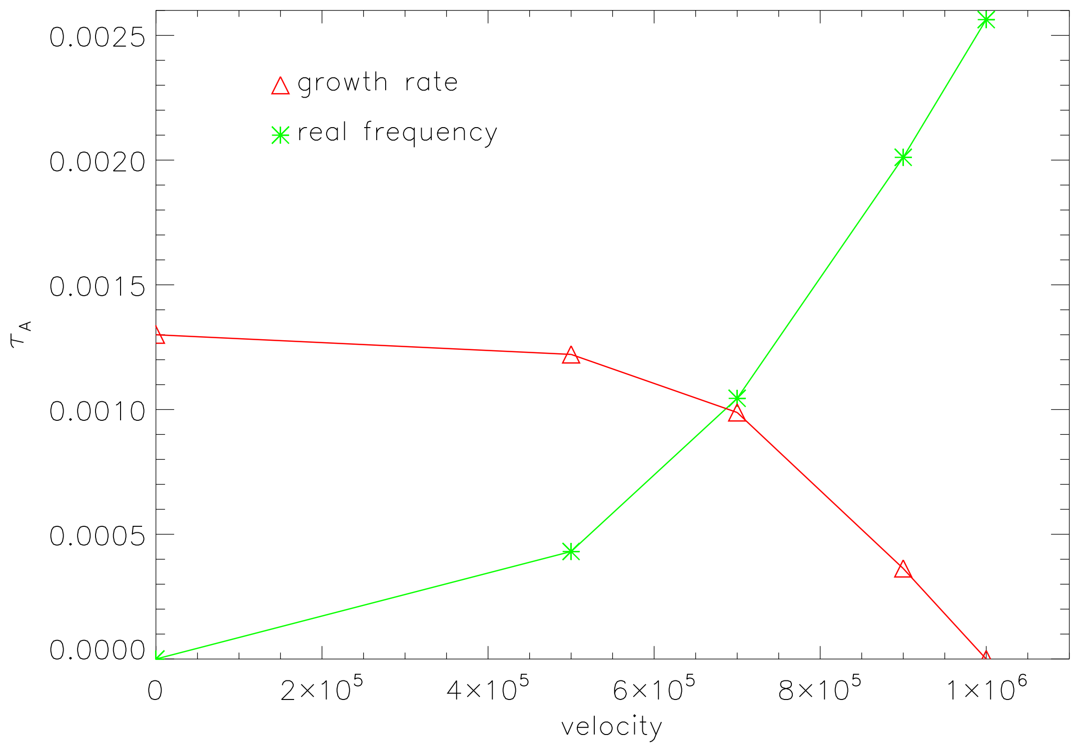
$$S = 1.e4,\ ka = 2,\ \gamma\tau_A = 1.3e-3$$





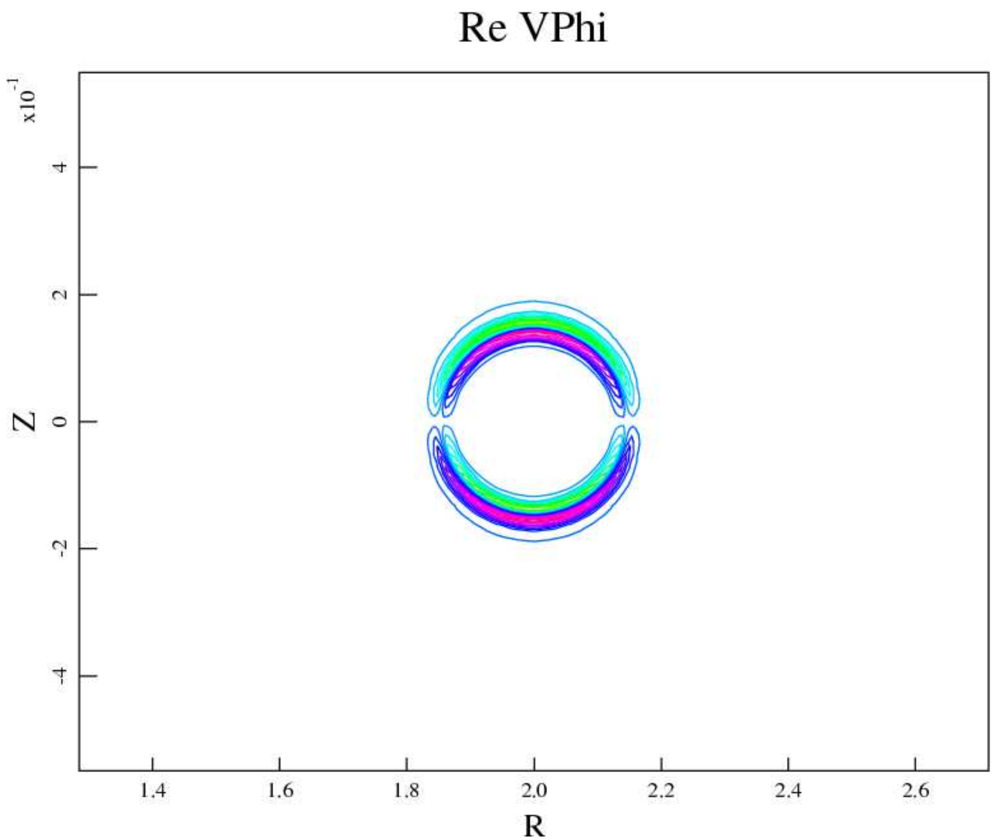
FLR Stabilization of RFP Tearing Mode

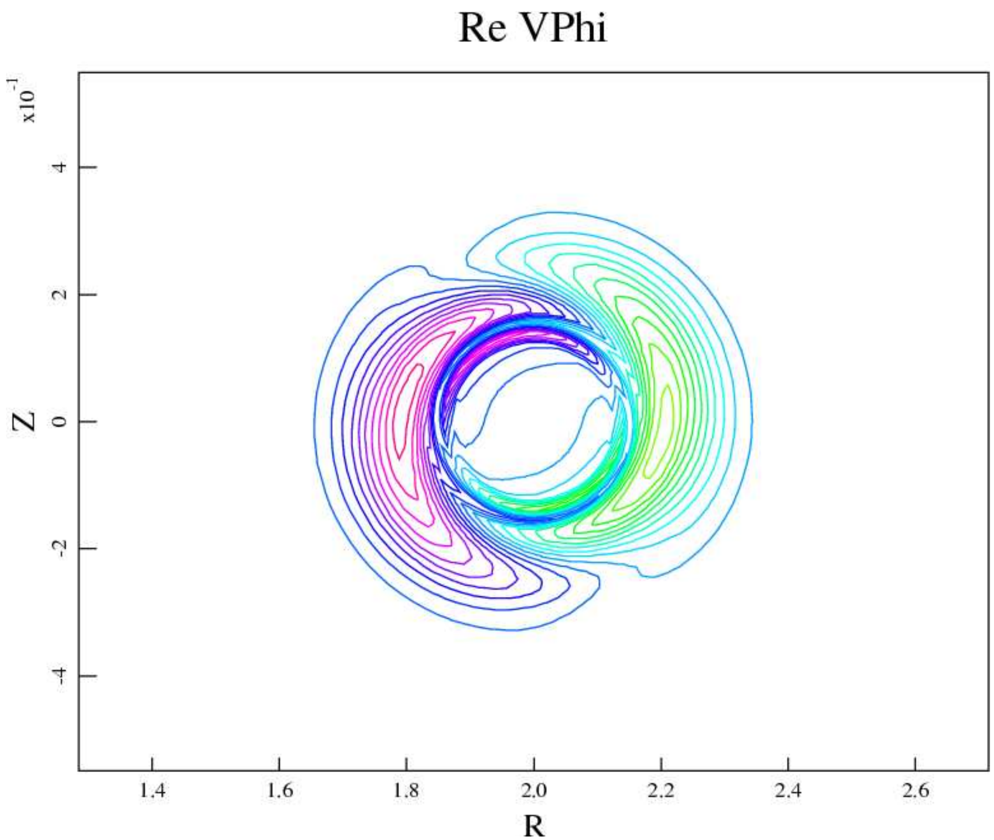
- initialize with monoenergetic particles, only $\mathbf{v} \times \delta\mathbf{B}$ in weight equation
- use **only** perpendicular pressure for comparison with theory



- stabilization at $\rho_h \simeq 4cm$
- simulation sees real frequency - probably due to finite spread in velocity

FLR Broadens Eigenmode Structure





A Step Back - Path to further development

- need improved particles for full pressure tensor
- to ease development move to rectangular mesh - slab geometry
- probably an easier problem - both physics and computation
- interesting problem in itself

Tearing mode in a slab

- strong guide field
- Gaussian current profile $J_\phi \propto \exp\left(-\frac{(x-x_0)^2}{a^2}\right)$
- two scale lengths of interest, a/ρ and $k_y\rho$
- effects of localization of energetic particles
- effect of finite $v_\parallel$
- distill out geometry

Stepping forward - Development plans

- move PIC-in-FEM to a CIC in nonuniform grid[a]
- use polynomial shape function $S \propto \left[1 - \left(\frac{r}{R}\right)^2\right]^\alpha$ where r is distance of particle from grid node, R is radius of influence, α is parameter to be explored
- compare polynomial shape function to present implementation
- apply full pressure tensor
- move to cylindrical geometry
  - term by term comparison with V. Svidzinski
- toroidal geometry
- apply to FRCs
- implement a full f PIC

[a] G. B. Jacobs and J. S. Hesthaven, 'High-order nodal discontinuous Galerkin PIC method on unstructured grids', *JCP*, **214**, 2006