

Growing MHD Equilibria with NIMROD - Initial LDX Simulations

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The Outline

- What do I mean by ‘Growing MHD Equilibria’
 - some NIMROD preliminaries
 - why not just interpolate some equilibria
- the how’s and what’s
 - the source term
 - some details
 - effects of diffusive terms
- some initial simulation results
- plans



NIMROD Preliminaries

- NIMROD is an initial value 3D XMHD code
- uses finite elements in two dimension, Fourier in the third
 - allows geometric flexibility
 - can handle extreme anisotropies, $\frac{\chi_{\parallel}}{\chi_{\perp}} \gg 10^6$
- semi-implicit advance, not restricted by magnetosonic CFL condition
 - model experiment relevant parameters, $S \sim 10^{7-8}$
- allows both linear and nonlinear simulations



Representation of NIMROD fields

- NIMROD fields use quadrilateral finite element-Fourier representation

$$\delta A(\mathbf{x}, t) = \sum_j A_{j,0}(t) \alpha_{j,0} + \sum_j \sum_n (A_{j,n}(t) \alpha_{j,n} + c.c.)$$

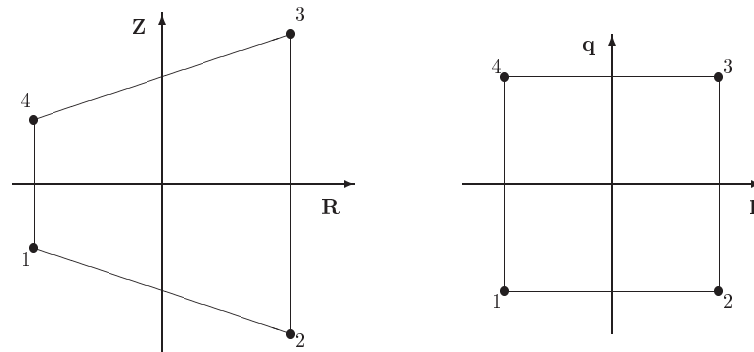
where

$$\alpha_{j,n} = N_j(p, q) \exp(in\phi)$$

(p, q) are logical coordinates, $N_j(p, q) = l_j(p)l_j(q)$ and

$$l_i(x) = \prod_{i=0, i \neq j}^k \frac{x - x_i}{x_j - x_i}$$

$k = pd + 1$, pd is the polynomial order of the Lagrange polynomial



NIMROD

- NIMROD's extended MHD equations

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} + \kappa_{divb} \nabla \nabla \cdot \mathbf{B}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\mathbf{E} = -\mathbf{U} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B}$$

$$+ \frac{m_e}{ne^2} \left[\sum_{\alpha} \frac{q_{\alpha}}{m_{\alpha}} (\nabla p_{\alpha} + \nabla \cdot \Pi_{\alpha}) \right] + \frac{m_e}{ne^2} \left[\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot (\mathbf{J}\mathbf{U} + \mathbf{U}\mathbf{J}) \right]$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{U}) = \nabla \cdot D\nabla n$$

$$mn \left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} \right) = \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \rho \nu \nabla \mathbf{V} - \nabla \cdot \Pi - \nabla \cdot p_h$$

$$\frac{n_{\alpha}}{\Gamma - 1} \left(\frac{\partial T_{\alpha}}{\partial t} + \mathbf{U}_{\alpha} \cdot \nabla T_{\alpha} \right) = -p_{\alpha} \nabla \cdot \mathbf{U}_{\alpha} - \nabla \cdot q_{\alpha} + Q_{\alpha} - \Pi_{\alpha} : \nabla \mathbf{U}_{\alpha}$$

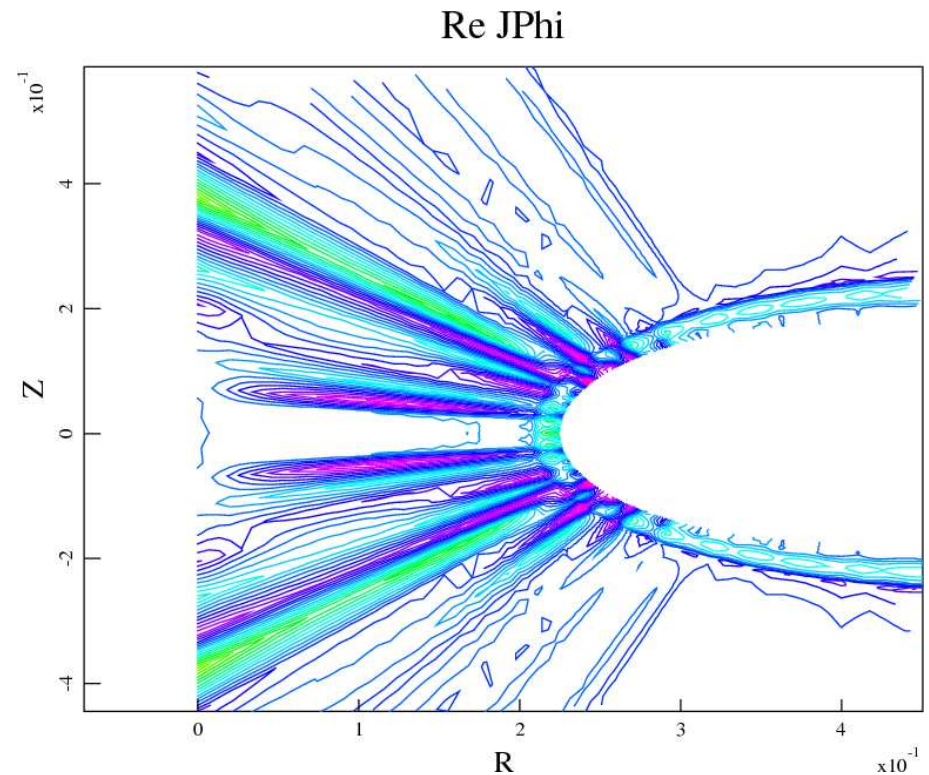
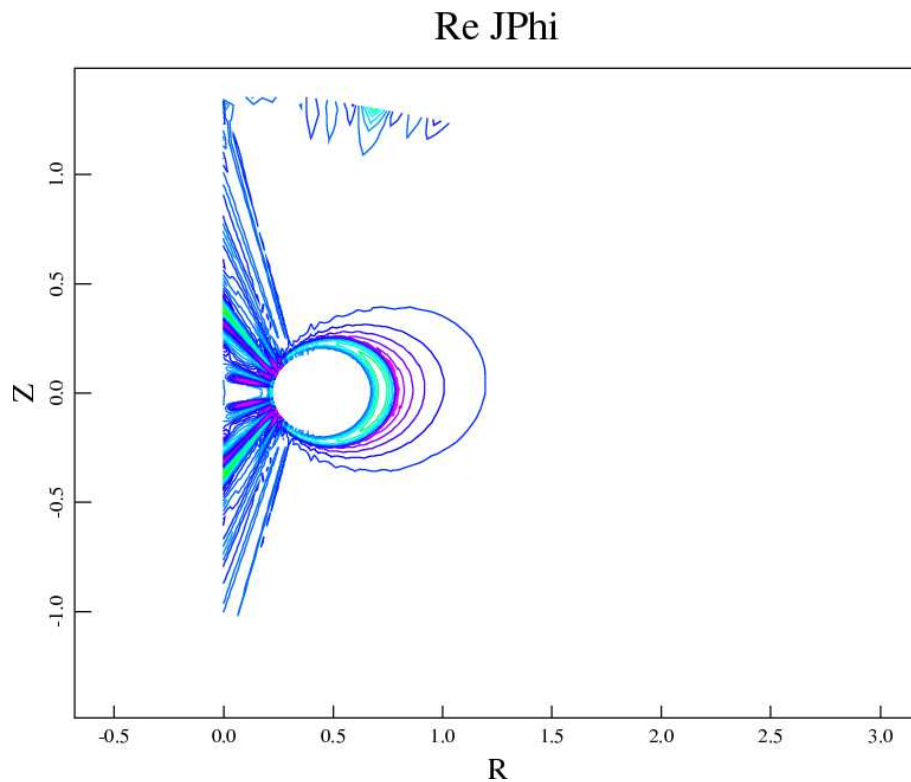


Why not interpolate some equilibria?

- **fluxgrid** is a set of routines that generates grid based on standard equilibria codes (e.g. EFIT)
 - sometimes NIMROD is finicky
 - sometimes equilibria are not good enough
 - may look into in the future
- instead try bicube interpolation to map fields onto NIMROD
 - use equilibria provided by D. Garnier
 - problems at $R = 0$
 - problems at cryodonut
 - involved a lot of ‘fiddling’ with the grid
 - not very successful



J_ϕ as the quality check



- relies on precise calculation of $\nabla \times$
- good test of Jacobian that relates (p, q) and (R, Z)

Heating a NIMROD plasma

- evolve only $n = 0$
- begin with a dipole field from a current inside cryodonut
- use heat source $Q \propto \exp\left(-\frac{(r - r_T)^2}{r_w^2}\right) \exp\left(-\frac{(z - z_T)^2}{z_w^2}\right)$
- $r_T \simeq .9m, r_w \simeq .3 - .5cm, z_T = 0., z_w = .2cm$ and $\chi_{\parallel} = 1 \times 10^6, \chi_{\perp} = 1$
- preheat helps
 - apply initial heating
 - move heat spot slightly outboard to finish
- initially relied on high number diffusivity and viscosity to maintain small density gradients
 - small density gradients keeps pressure contours better flux aligned
 - for simulations reduce diffusivities
 - results in transient ‘snap back’
 - let transient settle before beginning simulation



Show Movie

- <http://psi.psicenter.org/nelson/ldx/>

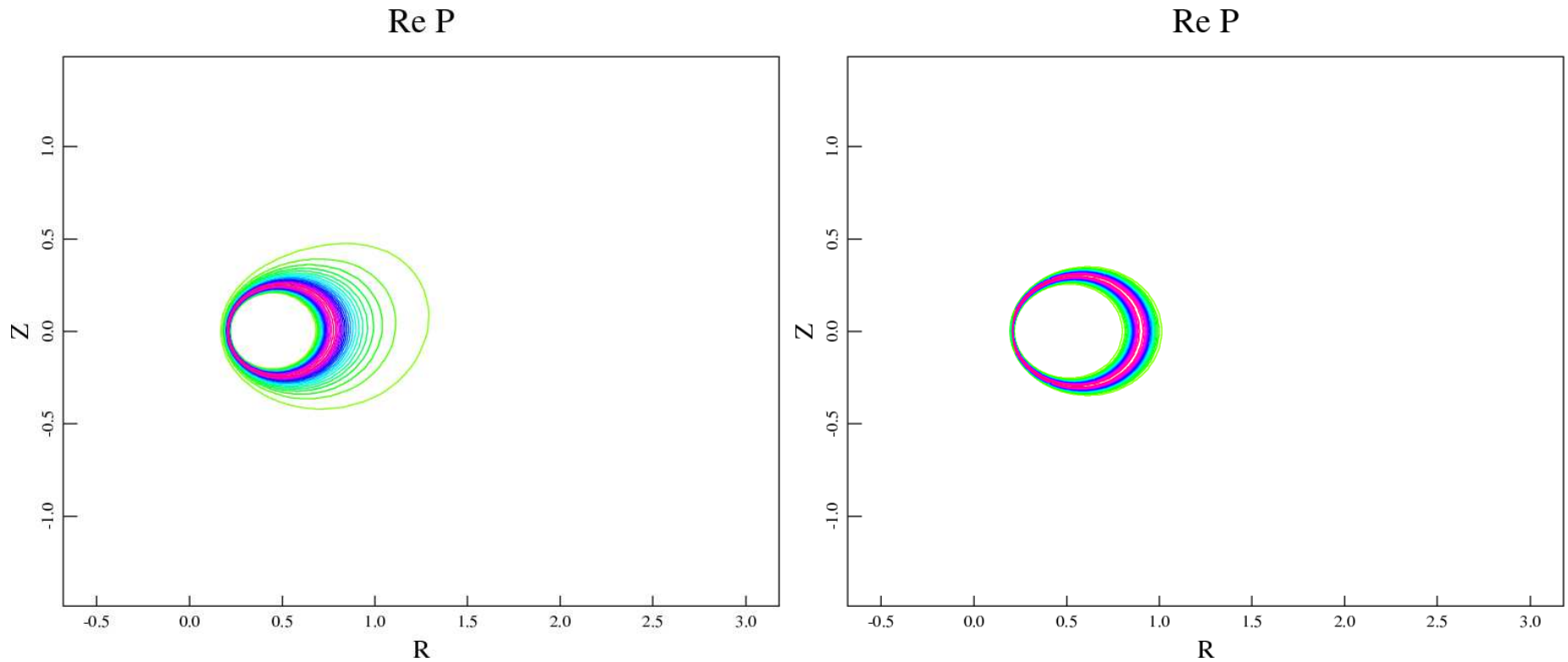


Diffusive Terms and some other issues

- (move to the NIMDEVEL version)
- use low resistivity to minimize flow without excessive viscosity or diffusivity
 - prevents ‘snap back’
 - having some instability problems in high field region
 - diffusive coefficients are probably too small
- use higher order polynomial
 - better thermal conduction - steeper profiles
 - switch to Lobatto nodes - better convergence properties for high order polynomials
 - better results faster (40x60x4 reduced to 24x48x5)
- more fiddling with the grid



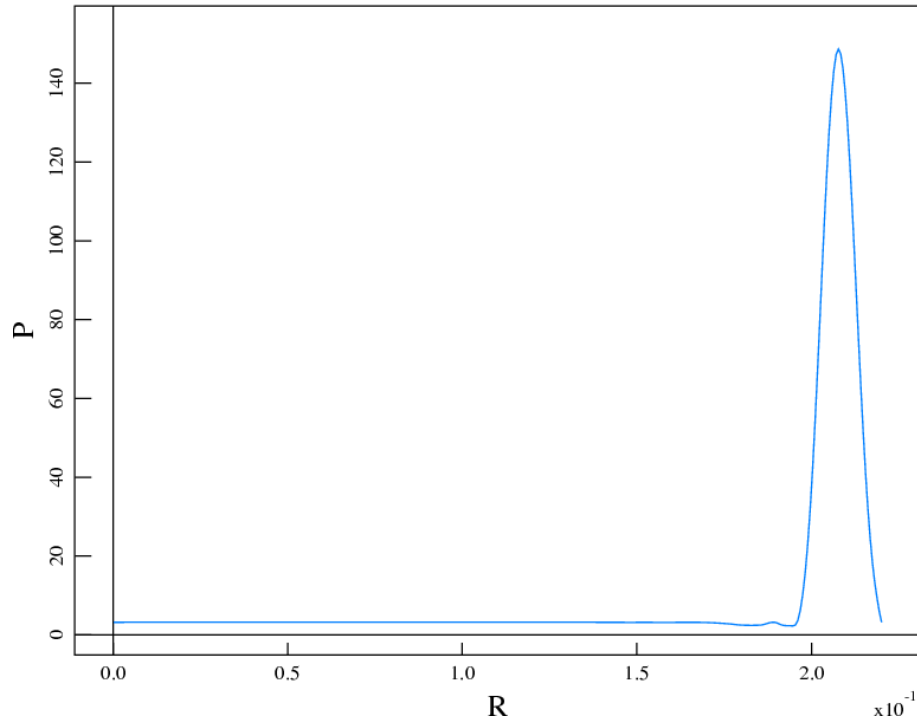
Pressure Contour Comparison



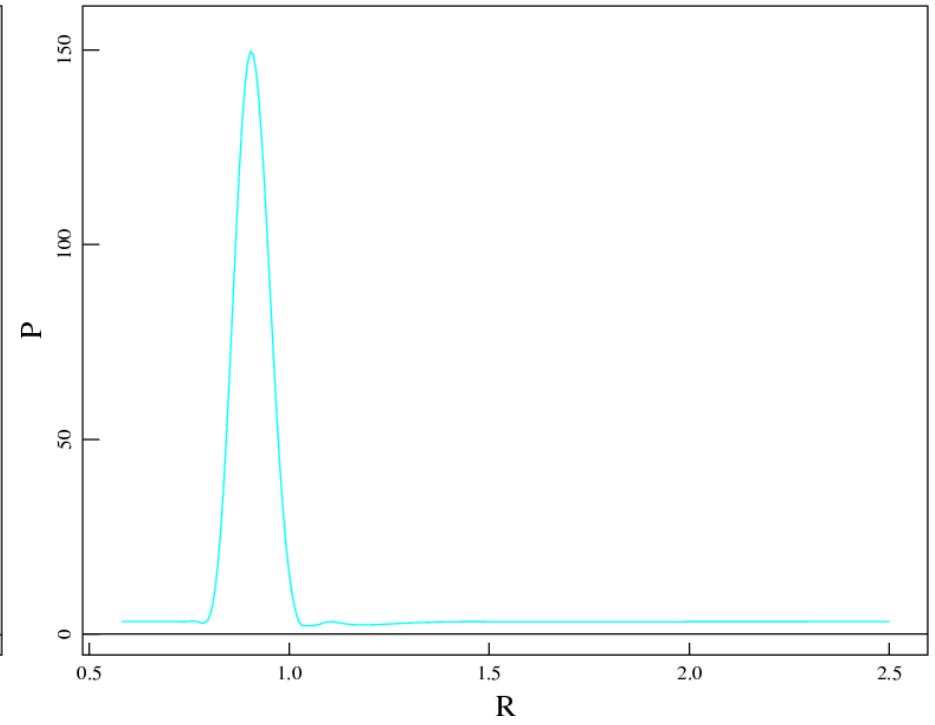
- pushing extremes of heating parameter space
- can 'grow' steep pressure profiles

Pressure Profile

Re P vs. R

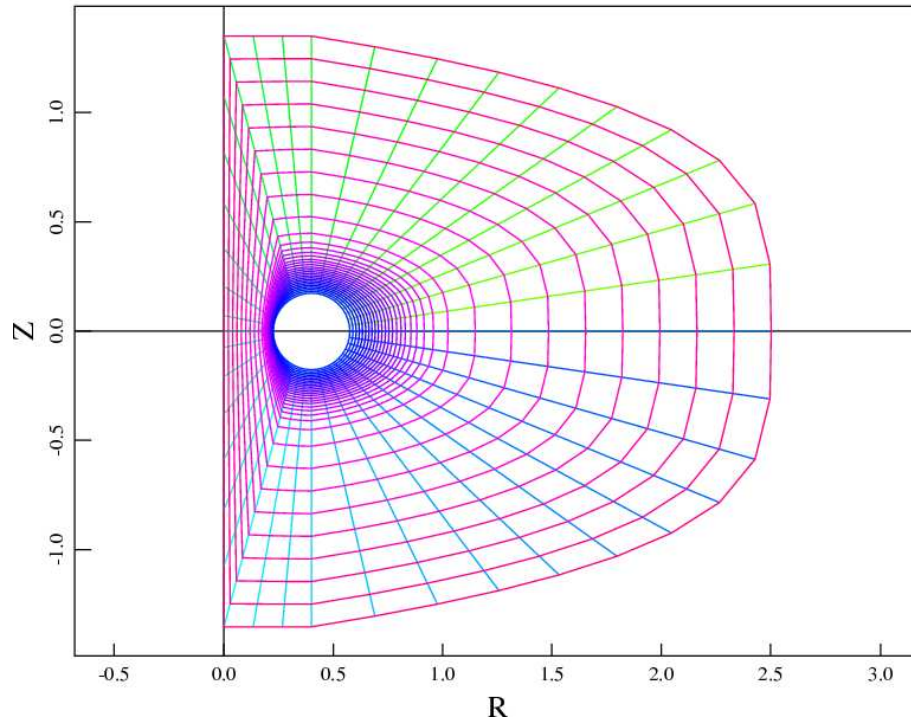


Re P vs. R

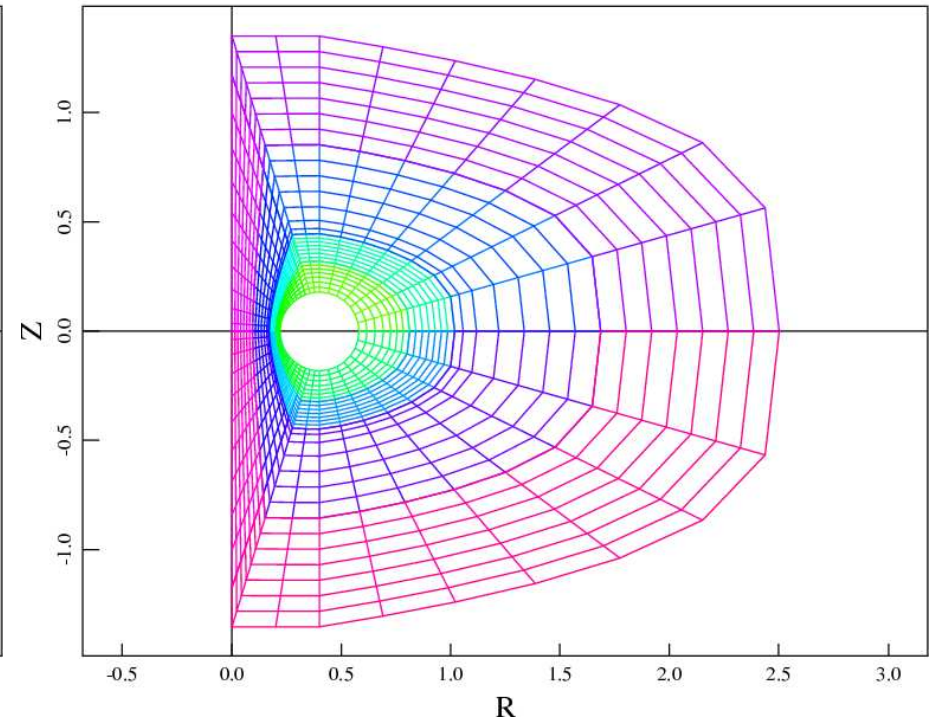


It all has to do with the grid (mostly)

Finite Element Mesh



Finite Element Mesh



- add some flexibility to the grid generation
- room for more improvements, particularly the crydonut

What's Next - Running a simulation

- with a sufficient 'equilibrium'
 - adjust diff. coef. to more experimental parameters
 - allow a settling phase
 - transfer profiles to multi modes
 - run
- add density source
- add localization to source
 - framework already exists in NIMDEVEL

