Preliminary Simulations of FLR effects on RFP tearing modes.

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Motivation and Background

- develop hybrid kinetic-MHD model with full orbit kinetics
- necessary to model ICC devices, e.g. spheromaks, RFP's, FRC's
- extensive efforts past and present - Barnes&Milroy, Park et al., Belova, Todo
- leverage successful extended MHD capabilities of NIMROD [a] [b]
- extend drift kinetic hybrid work
- test full kinetic model in stabilization of RFP tearing mode - V. Svidzinski PoP 2004
- demonstrate capabilities of simulation
- preliminary steps towards ultimate simulation

[a] C. R. Sovinec, et al., "Nonlinear magnetohydrodynamic simulations using higher-order finite elements", JCP **195**, 355 (2004)

[b] D. D. Schnack, et al., "Computational modeling of fully ionized magnetized plasmas using the fluid approximation", PoP **13**, 058103 (2006)

Outline of Talk

- model equations
  - Extended MHD fluid equations
  - δf and the kinetic equations
  - hybrid kinetic-MHD equation
- comparison to drift kinetic results
- RFP model and summary of Svidzinski's theoretic results
- simulation results
- conclusion and future directions

NIMROD

- NIMROD evolves the extended MHD equations

$$
\begin{aligned}
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla \times \mathbf{E} + \kappa_{divb} \nabla\nabla \cdot \mathbf{B} \\
\nabla \times \mathbf{B} &= \mu_0 \mathbf{J} \\
\mathbf{E} &= -\mathbf{U} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} \\
+\frac{m_e}{ne^2}\left[\sum_\alpha \frac{q_\alpha}{m_\alpha}(\nabla p_\alpha + \nabla \cdot \Pi_\alpha)\right] &+ \frac{m_e}{ne^2}\left[\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot (\mathbf{JU} + \mathbf{UJ})\right] \\
\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{U}) &= \nabla \cdot D \nabla n \\
mn\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U}\right) &= \mathbf{J} \times \mathbf{B} - \nabla p + \nabla \cdot \rho\nu\nabla\mathbf{V} - \nabla \cdot \Pi - \nabla \cdot p_h \\
\frac{n_\alpha}{\Gamma - 1}\left(\frac{\partial T_\alpha}{\partial t} + \mathbf{U}_\alpha \cdot \nabla T_\alpha\right) &= -p_\alpha \nabla \cdot \mathbf{U}_\alpha - \nabla \cdot q_\alpha + Q_\alpha - \Pi_\alpha : \nabla \mathbf{U}_\alpha
\end{aligned}
$$

- finite element in one plane, pseudo-spectral in 3rd direction

Summary of the δf PIC Method [a] [b]

- PIC is a Lagrangian simulation of phase space $f(\mathbf{x}, \mathbf{v}) \rightarrow f(\mathbf{x}(t), \mathbf{v}(t))$
  - discretize $f(\mathbf{x}, \mathbf{v})$ - sample with 'particles' or markers
  - equations of motion are used to advance 'particles'
  - spatial grid is used to compute moments of f
- in principle, $f(\mathbf{x}, \mathbf{v})$ contains everything
- PIC is noisy, can't beat $1/\sqrt{N}$
- δf PIC reduces the discrete particle noise associated with conventional PIC

[a] S. E. Parker and W. W. Lee, 'A fully nonlinear characteristic method for gyro-kinetic simulation', *Physics of Fluids B*, **5**, 1993

[b] G. Hu and J. A. Krommes, "Generalized weighting scheme for δf particle simulation method", *Physics of Plasmas*, **1**, 1994

The δf PIC Method cont.

- begin with the Vlasov Equation

$$\frac{\partial f(\mathbf{z})}{\partial t} + \dot{\mathbf{z}} \cdot \frac{\partial f(\mathbf{z})}{\partial \mathbf{z}} = 0$$

- split phase space distribution into steady state and evolving perturbation:
  - $f = f_{eq}(\mathbf{z}) + \delta f(\mathbf{z}, t)$
  - moments of f_{eq} easy to compute

- δf evolves along the characteristics $\dot{\mathbf{z}}$ (control variates MC[a])

$$\dot{\delta f} = -\tilde{\mathbf{z}} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}}$$

  using $\mathbf{z} = \mathbf{z}_{eq} + \tilde{\mathbf{z}}$ and $\dot{\mathbf{z}}_{eq} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}} = 0$

- must choose a stationary f_{eq}, often chosen as a function of canonical variables

[a] A. Y. Aydemir, "A unified MC interpretation of particle simulations...", *Physics of Plasmas*, **1**, 1994

δf and the Lorentz Equations

- Lorentz equation of motion

$$\dot{\mathbf{x}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = \frac{q}{m}(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

- for full kinetic equations use[a]

$$f_0 = f(\mathbf{x}, v^2) + \frac{1}{\omega_c}(\mathbf{v} \cdot \mathbf{b} \times \nabla f)$$

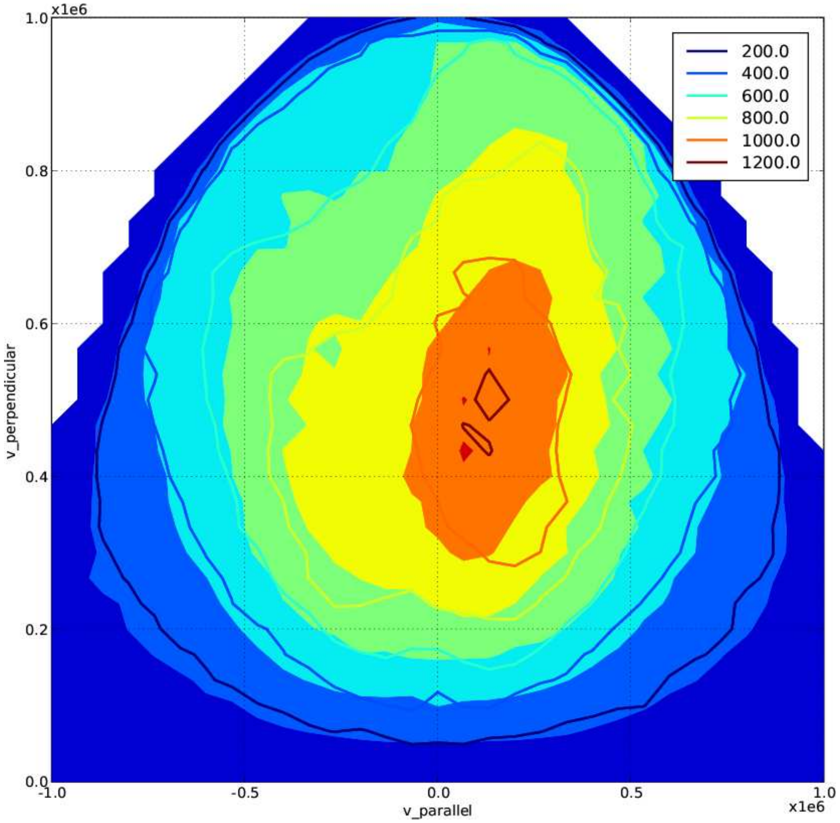
- weight equation is

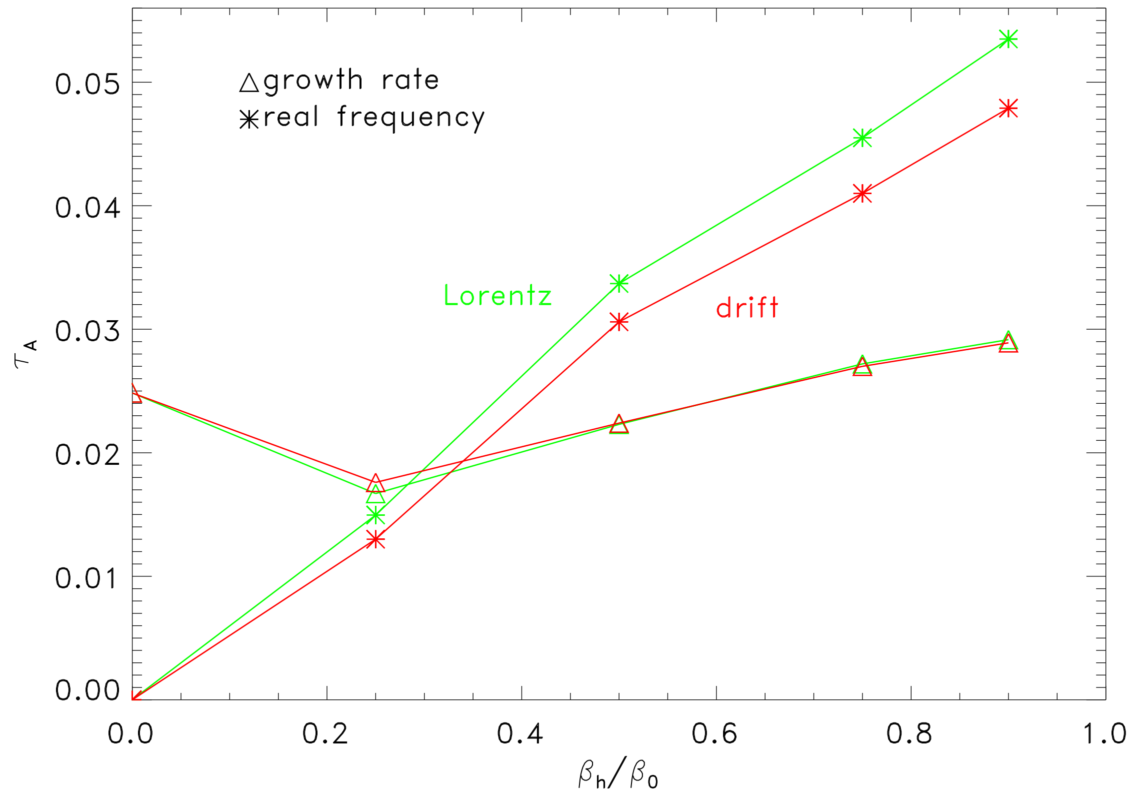
$$\dot{\delta f} = -\frac{\delta\mathbf{E} + \mathbf{v} \times \delta\mathbf{B}}{B} \cdot \mathbf{b} \times \nabla f - \frac{2q}{m}\delta\mathbf{E} \cdot \mathbf{v}\frac{\partial f}{\partial v^2}$$

- satisfying in that it is very similar to drift kinetic weight equation

[a] M. N. Rosenbluth and N. Rostoker "Theoretical Structure of Plasma Equations", Physics of Fluids **2** 23 (1959)

Boris Algorithm Nicely Recovers Drift Kinetic Results



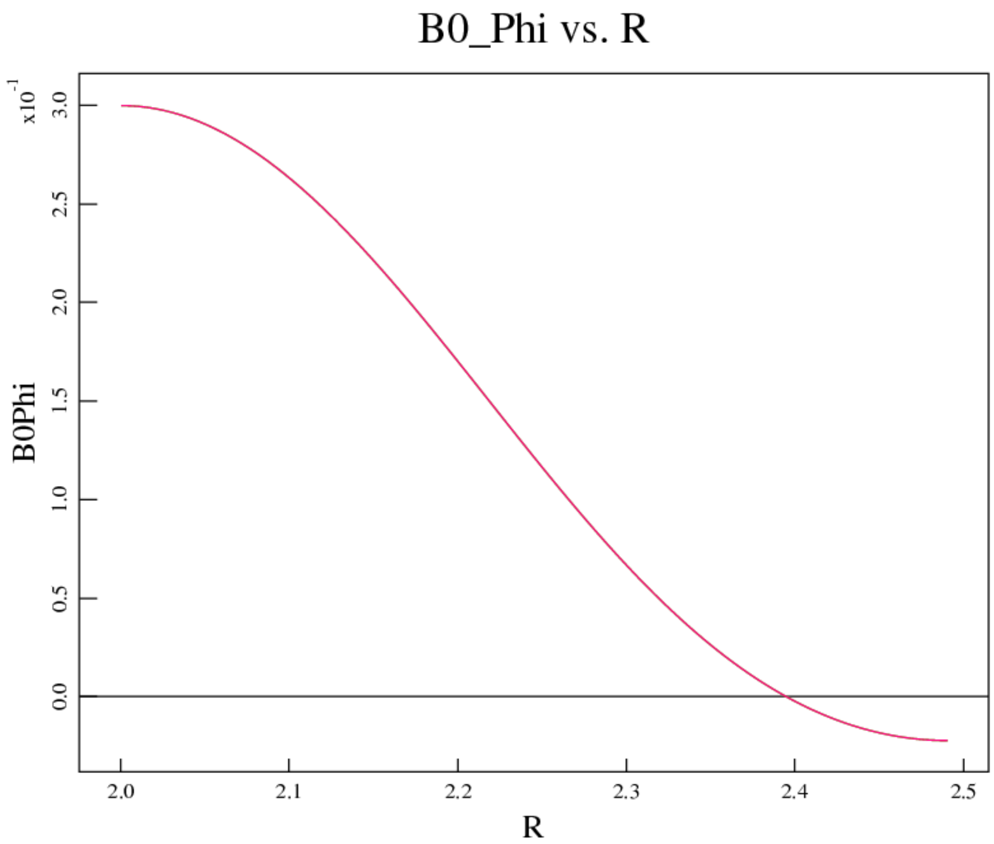


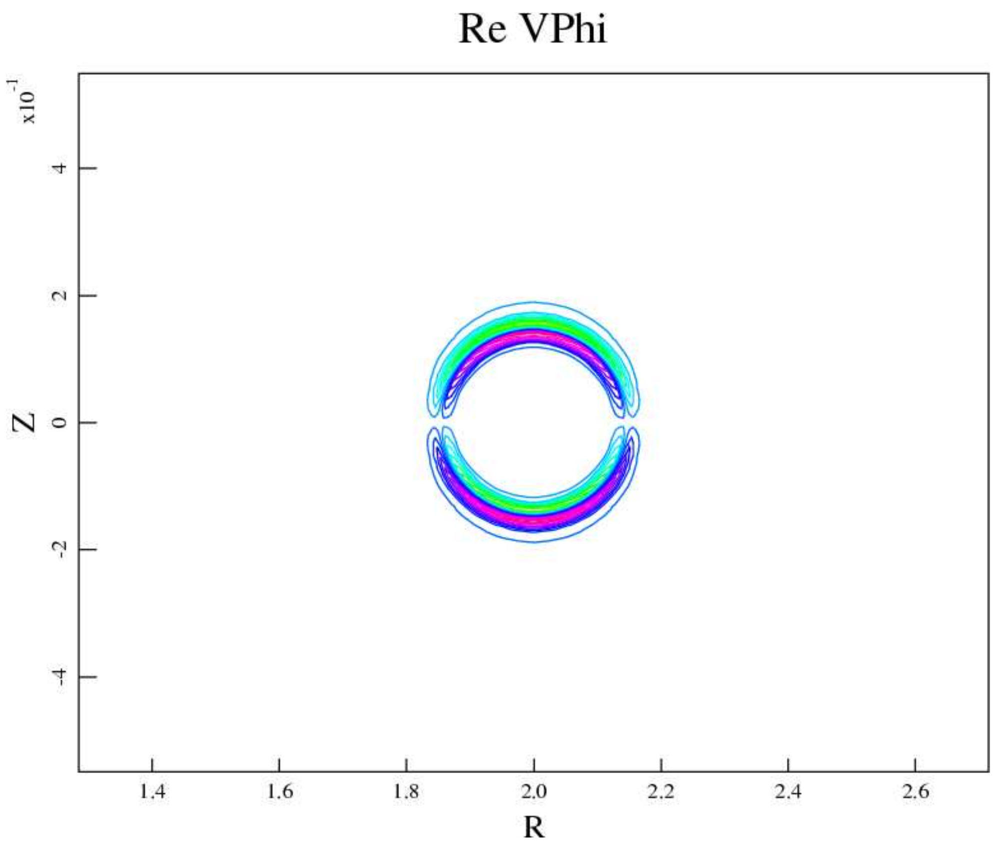
- velocity space overlap - infer drift trajectories are recovered by Lorentz push
- reproduce linear $(1, 1)$ kink mode stabilization by hot particles

Linear Simulations of Tearing Modes in a RFP

- alpha model equilibrium $\nabla \times \mathbf{B} = \mu \mathbf{B} \quad \mu = 2\Theta \left[1 - \left(\frac{r}{a} \right)^{\alpha_0} \right]$
- parameters for straight cylinder

$$a = .5m,\ B_0 = .3T,\ \Theta = 1.75,\ \alpha_0 = 3,$$
$$S = 1.e4,\ ka = 2,\ \gamma\tau_A = 1.3e-3$$





Summary of Theoretic Prediction of V. Svidzinski[a]

- calculates dielectric response of energetic particles for $\mathbf{E}, \mathbf{B}$
- uses conductivity tensor $\sigma^{\mathbf{E}}, \sigma^{\mathbf{B}}$ to calculate $\mathbf{J}_h$ contribution from energetic particles
- focuses on $\sigma^{\mathbf{E}}_{yy}$ and $\sigma^{\mathbf{B}}_{xy}$
- uses Ampere's relation $\nabla \times \mathbf{B} = \mathbf{J} + \mathbf{J}_h$
- treats the energetic particles as a perturbation to $\mathbf{B} = \tilde{\mathbf{B}} + \mathbf{B}_h$ to solve the tearing layer problem in RFP
- uses $f(\mathbf{v}) \propto \delta(v_\perp - v_0)$
- show increasing stabilization with increasing ρ/a

[a] V. A. Svidzinski and S. C. Prager, "Effects of particles with large gyroradii on resistive magnetohydrodynamic stability", PoP **11** 980, 2004

Simulation Details

- Boris push with orbit averaging to accomodates disparate time scales
- energetic ion density profile $\propto \exp\left[-\left(\frac{r}{0.45a}\right)^2\right]$
- use hybrid kinetic-MHD equations[a]

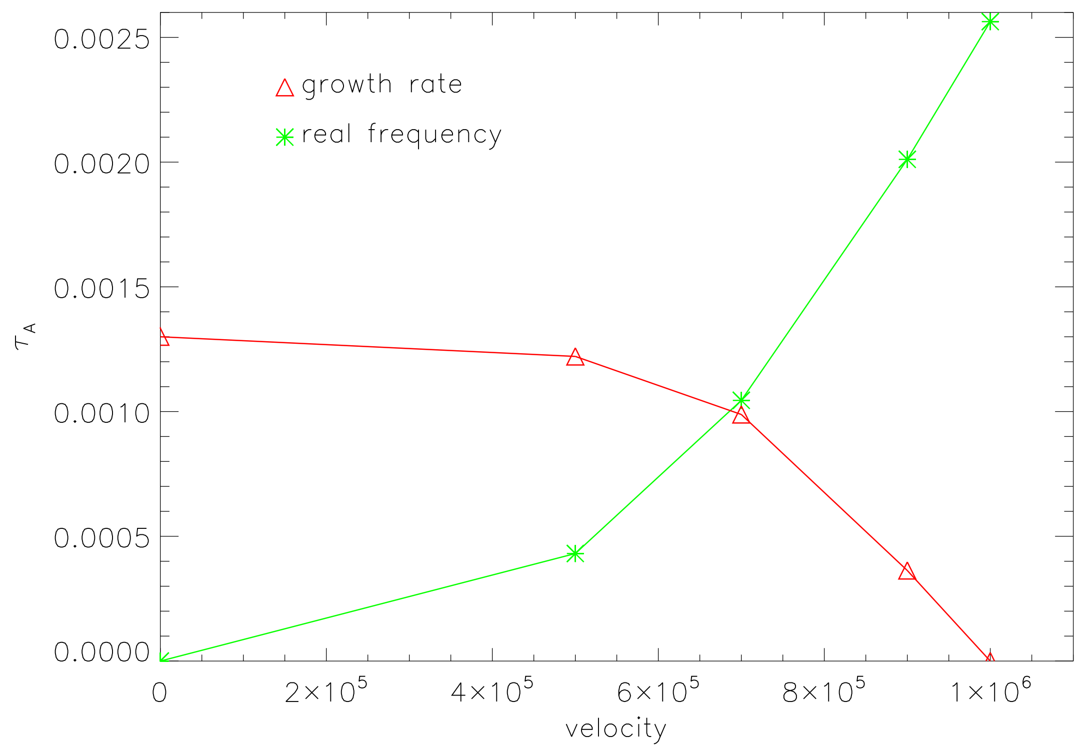
$$\rho\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U}\right) = \mathbf{J}\times\mathbf{B} - \nabla\cdot\underline{\mathbf{p}}_b - \nabla\cdot\underline{\mathbf{p}}_h$$

- use CGL pressure tensor $\delta\underline{\mathbf{p}}_h = \begin{pmatrix} p_\perp & 0 & 0 \\ 0 & p_\perp & 0 \\ 0 & 0 & p_\parallel \end{pmatrix}$

[a] C.Z.Cheng, "A Kinetic MHD Model for Low Frequency Phenomena", *J. Geophys. Rev*, **96**, 1991

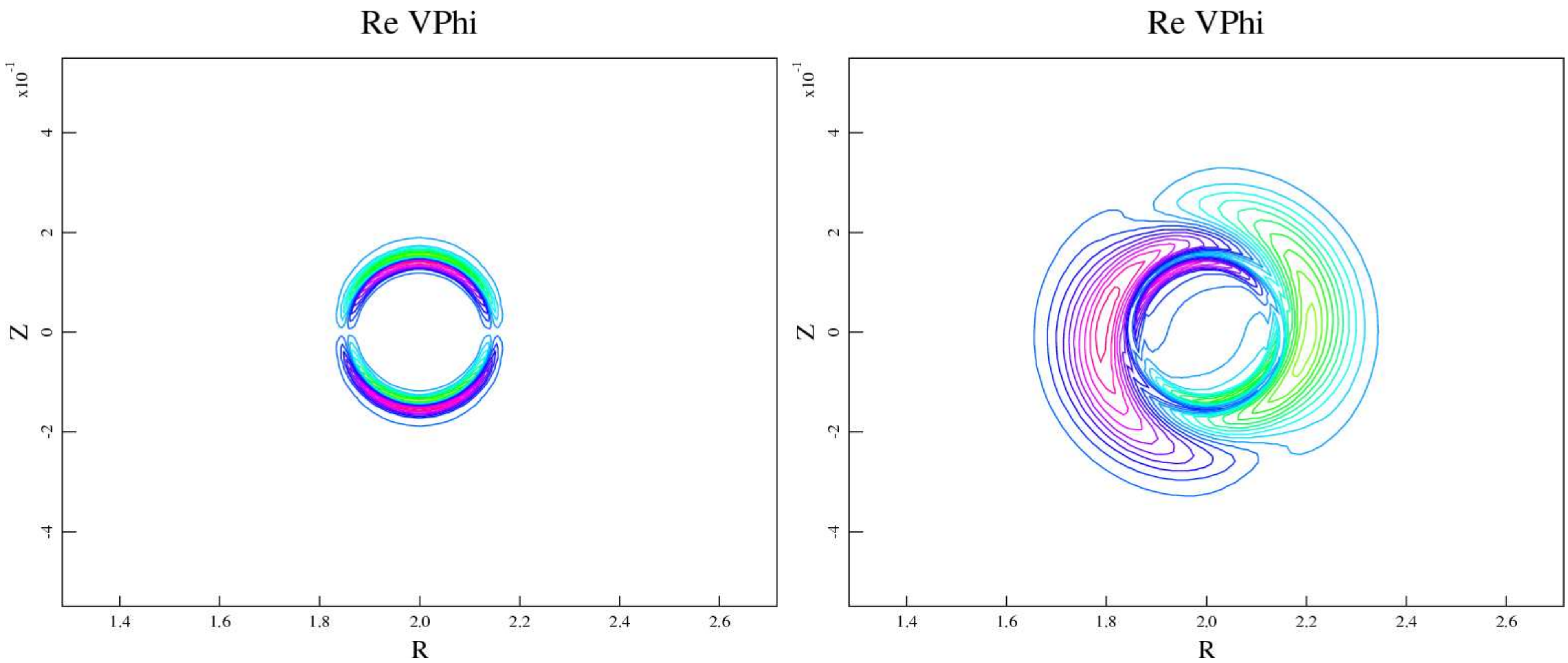
FLR Stabilization of RFP Tearing Mode

- initialize with monoenergetic particles, only $\mathbf{v} \times \delta\mathbf{B}$ in weight equation
- use **only** perpendicular pressure for comparison with theory



- stabilization at $\rho_h \simeq 4cm$
- simulation sees real frequency - probably due to finite spread in velocity

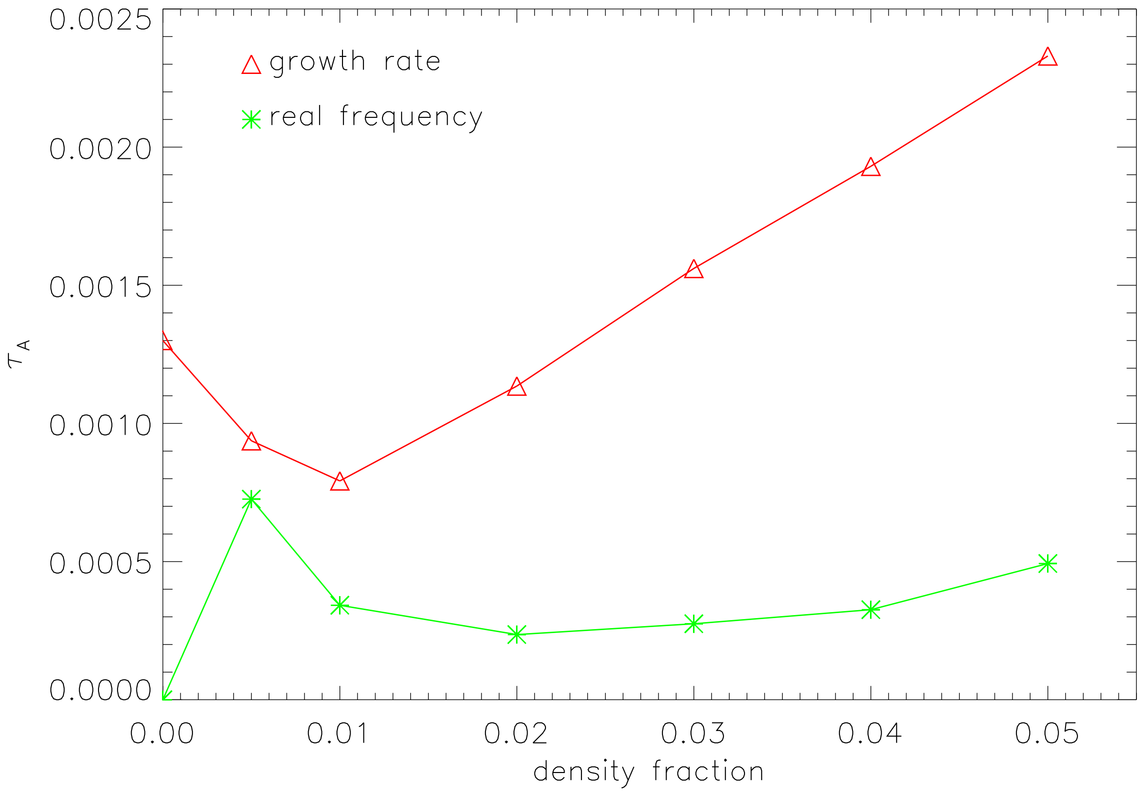
FLR Broadens Eigenmode Structure



- encouraging results → turn on more physics

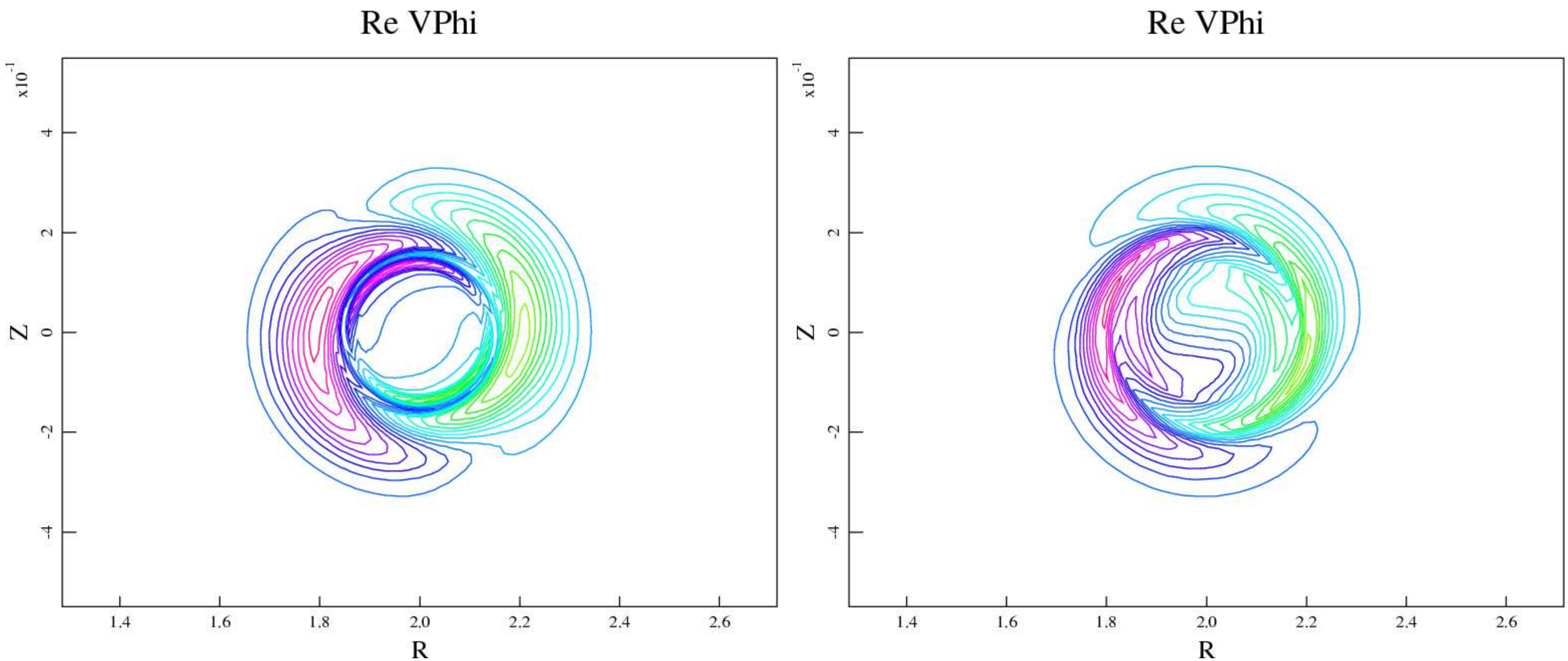
?!Excitation of FLR driven Instability!?

- scan in energetic density
- $\delta\mathbf{E} + \mathbf{v} \times \delta\mathbf{B}$ in weight equation
- both parallel and perpendicular pressure



- dramatically changes "mode-ology "

?!FLR Eigenmode!?



- no semblence of original tearing mode structure

End Remarks

- develop hybrid kinetic-MHD model with full orbit kinetics
- compares well to drift kinetic results
- agrees with predictions of Svidzinski's theoretic results
- hints at interesting 'mode-ology' with inclusion of more kinetic physics
- work continues on extending the model and algorithm
- stay tuned