Extension of Drift Kinetic Hot Particles to Full Orbits in NIMROD

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Background and Motivation

- develop hybrid kinetic-MHD model with full orbit kinetics
- necessary to model ICC devices, e.g. spheromaks, RFP's, FRC's
- extensive efforts past and present - Barnes&Milroy, Park et al, Belova, Todo
- leverage successful extended MHD capabilities of NIMROD[a]
- extend drift kinetic hybrid work
- also gives a point of contact and reality check
- first test of full kinetic model is stabilization of tearing RFP tearing mode - V. Svidzinski PoP 2004
- preliminary steps towards ultimate simulation

[a]D. D. Schnack, et al, "Computational modeling of fully ionized magnetized plasmas using the fluid approximation", PoP **13**, 058103 (2006)

Full Kinetic Equations

- Lorentz equation of motion

$$\dot{\mathbf{x}} = \mathbf{v} \quad \dot{\mathbf{v}} = \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \tag{1}$$

- for full kinetic equations use[a]

$$f_0 = f(\mathbf{x}, v^2) + \frac{1}{\omega_c}\left(\mathbf{v} \cdot \mathbf{b} \times \nabla f\right) \tag{2}$$

- weight equation is

$$\dot{\delta f} = -\frac{\delta \mathbf{E} + \mathbf{v} \times \delta \mathbf{B}}{B} \cdot \hat{\mathbf{b}} \times \nabla f - \frac{q}{m}\delta \mathbf{E} \cdot \frac{\partial f}{\partial \mathbf{v}} \tag{3}$$

- recovers dominant terms of drift kinetic weight equation - $\mathbf{v}_{E\times B}$ and $v_{\|}$
- in the limit $n_h \ll n_0$, $\beta_h \sim \beta_0$

$$\rho\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U}\right) = \mathbf{J} \times \mathbf{B} - \nabla \cdot \underline{\mathbf{p}}_b - \nabla \cdot \underline{\mathbf{p}}_h$$

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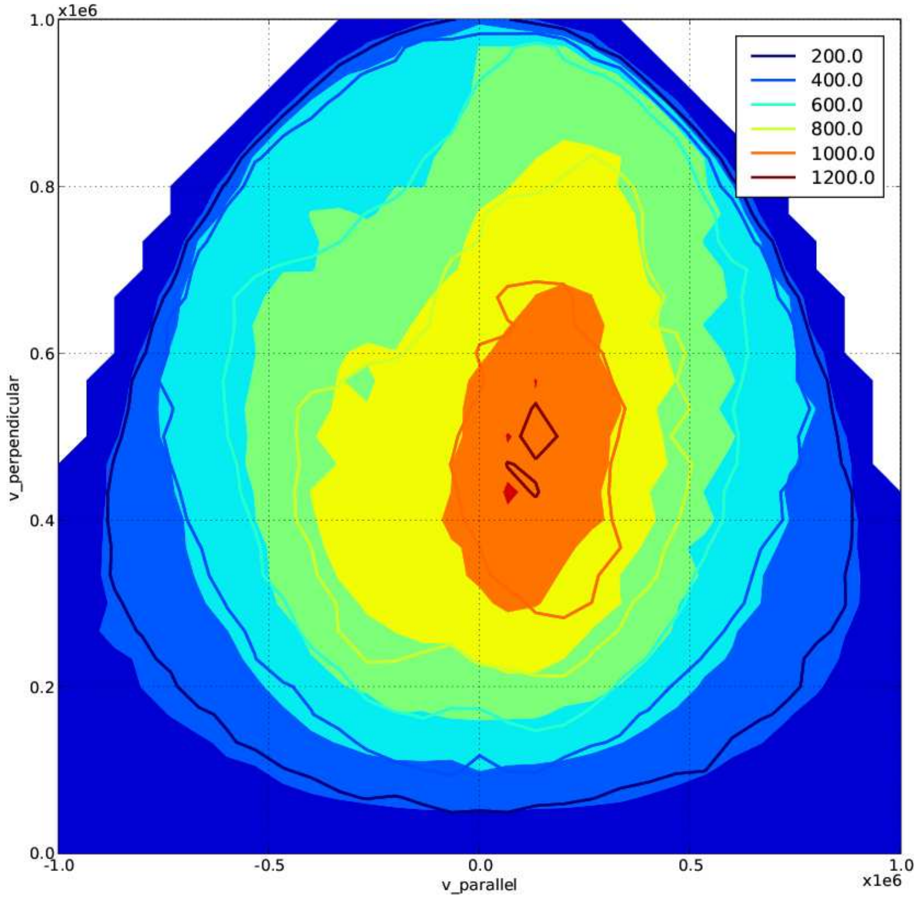
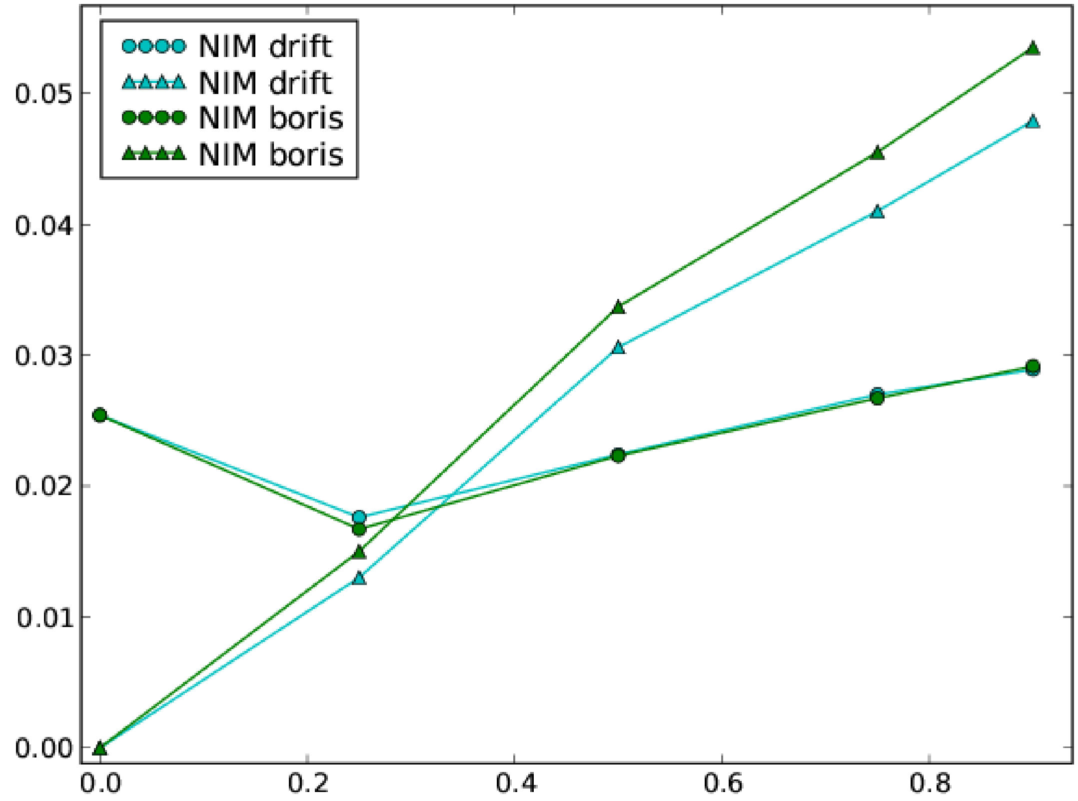
[a] M. N. Rosenbluth and N. Rostoker "Theoretical Structure of Plasma Equations", Physics of Fluids vol**2** 23 (1959)

Boris Algorithm nicely recovers Drift Kinetic results





- velocity space overlap - infer drift trajectories are recovered by Lorentz push
- reproduce linear $(1,1)$ kink mode stabilization by hot particles

Summary of Theoretic Prediction of V. Svidzinski[a]

- calculates dielectric response of energetic particles for $\mathbf{E}, \mathbf{B}$
- uses conductivity tensor $\sigma^{\mathbf{E}}, \sigma^{\mathbf{B}}$ to calculate $\mathbf{J}_h$ contribution from energetic particles
- uses Ampere's relation $\nabla \times \mathbf{B} = \mathbf{J} + \mathbf{J}_h$
- treats the energetic particles as a perturbation to $\mathbf{B} = \tilde{\mathbf{B}} + \mathbf{B}_h$ to solve the tearing layer problem in RFP
- uses $f(\mathbf{v}) \propto \delta(v_\perp - v_0) \exp(-v_\parallel^2/v_T^2)$, $v_T = 0$
- shows increasing stabilization with increasing ρ/a

[a] V. A. Svidzinski and S. C. Prager, "Effects of particles with large gyroradii on resistive magnetohydrodynamic stability", PoP **11** 980, 2004

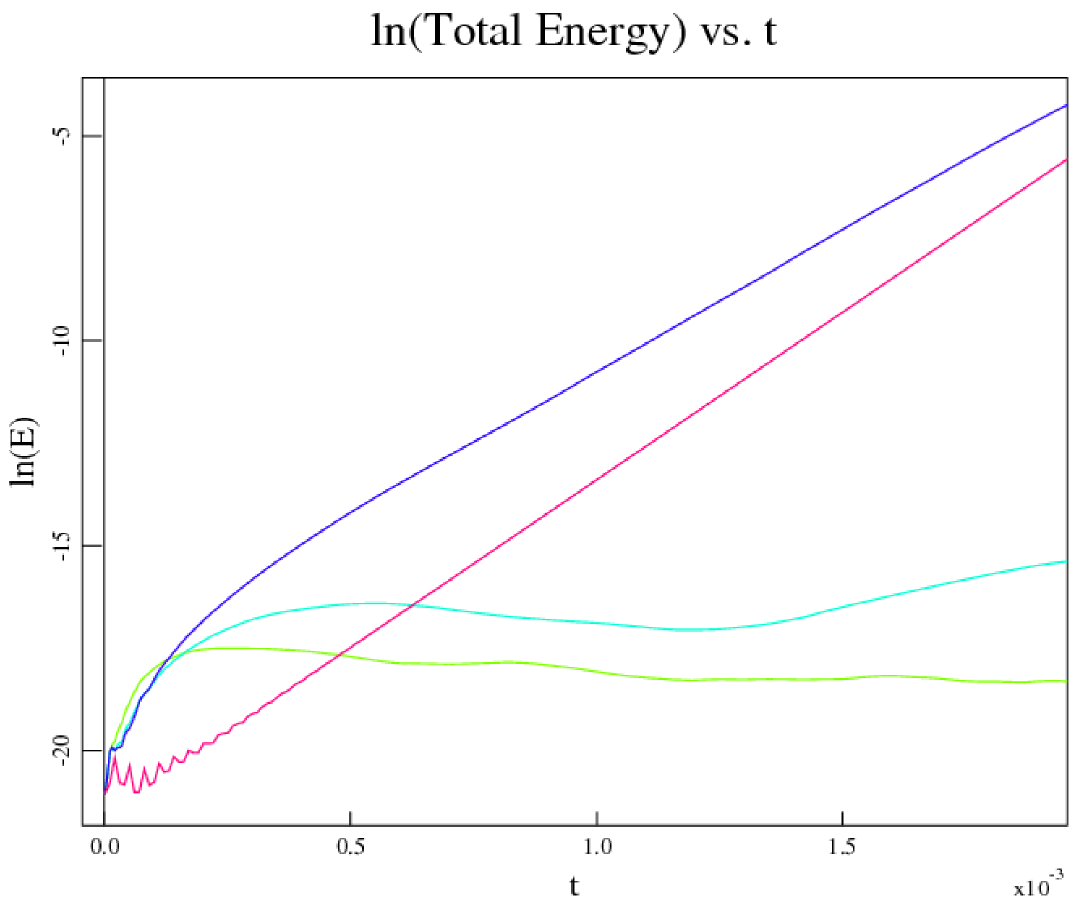
Simulation Details

- using δf PIC[a]
- Boris push with orbit averaging
- use only $\dfrac{\delta\mathbf{E} + \mathbf{v}\times\delta\mathbf{B}}{B}\cdot\hat{\mathbf{b}}\times\nabla f$ in weight equation
- equilibrium current profile $\mu = 2\Theta\left[1 - \left(\dfrac{r}{a}\right)^{\alpha_0}\right]$
- energetic ion density profile $\propto \exp\left[-\left(\frac{r}{0.45a}\right)^2\right]$
- in simulation load initial $\delta(v_\perp - v_0)$
- dynamics results in finite spread in $(v_\perp, v_\parallel)$
- subtract off $v_\parallel$ in calculating weight for $\mathbf{v}\times\delta\mathbf{B}$ comparison

[a] S. E. Parker and W. W. Lee, "A fully nonlinear characteristic method for gyrokinetic simulation", PFB **5**, 77 (1993)

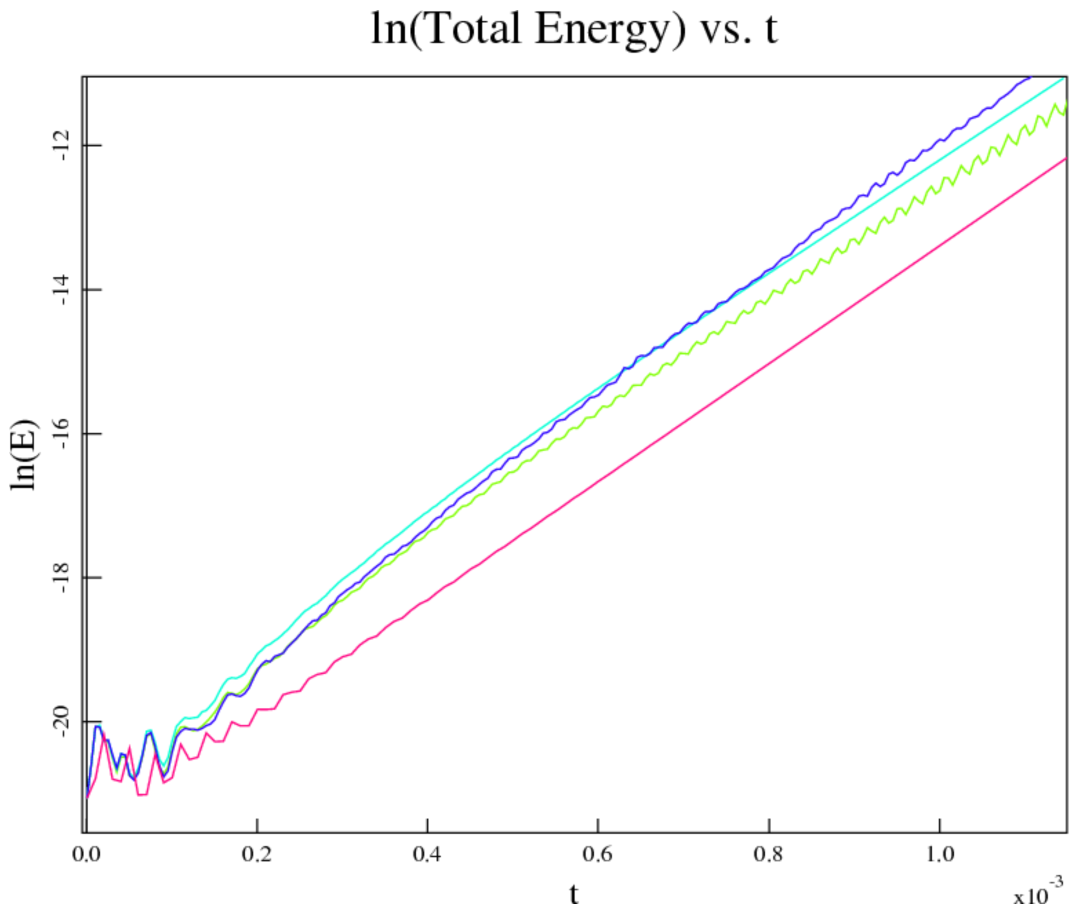
FLR Stabilization of RFP Tearing Mode

- current profile $\Theta = 1.75, \alpha_0 = 3, S = 10^4$ has $\gamma\tau_A = 1.1e - 3$
- examine effect of increasing Larmour radius $2\rho_i/a = [.15, .21, .26]$ equivalent to $v = [1.e6, 1.4e6, 1.8e6]$



- $\mathbf{v} \times \delta\mathbf{B}$ shows strong stabilizing in agreement with theory

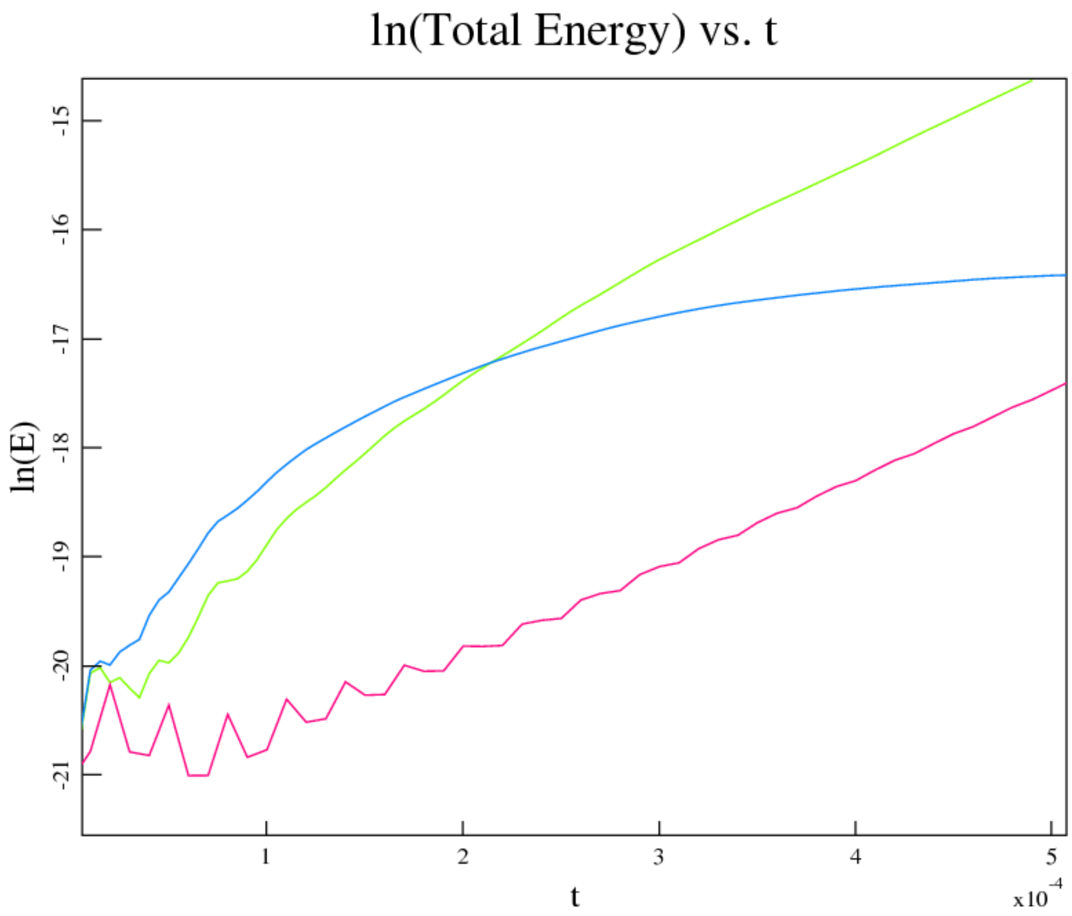
- $\delta\mathbf{E}$ negligible effect



- V.Svidzinski points out that finite $v_{\|}$ distribution may reduce effect through phase mixing

Effects of Velocity Distribution

- Maxwellian distribution with $v_T = 1.4e6$
- preliminary result from both terms in weight equation



- neglible stabilizing effect

Summary

- full kinetic particles are implemented and running
- compares well with drift kinetic kink
- shows qualitative agreement with FLR stabilization of tearing mode
- initial runs show that distribution in velocity decreases (possibly eliminates) effect
- preliminary simulations - more analysis needed before conclusions can be drawn
- progress continues - stay tuned