Quarterly Progress Report of the PSI-Center (July - September 2009)

by

Thomas Jarboe, Alan Glasser, Richard Milroy, Brian Nelson,
Uri Shumlak, and Carl Sovinec

The Plasma Science and Innovation Center (PSI-Center) has accomplished a great deal this quarter. The PSI-Center is organized into four groups: Boundary Conditions and Geometry, Two-Fluid and Transport, FLR and Kinetic Effects, and Interfacing. Each group has made good progress and the results from each group are given in detail.

Progress Report for the BC&G Group (*U. Shumlak, W. Lowrie, G. Marklin, and E. Meier*)

Accomplishments

- Prepared a manuscript about mesh deformation metrics applied to high-order finite (spectral) elements. The manuscript will be further edited and submitted for publication to the Journal of Computational Physics (or similar journal).
- Continued investigation of mesh quality metrics with several distorted grids using the MHD equations. A slow magnetosonic wave was initialized and used as a test problem with a known solution for comparing to the quality of the mesh.
- Updated the CUBIT mesh generator/CAD interface to the SEL/HiFi code for reading grids with an arbitrary number of blocks. This is for use with the multiblock implementation of the code, which is currently being developed.
- The first iteration of the multiblock development is near completion. This first step allows for an arbitrary number of structured blocks connected in a structured fashion. This iteration will allow more flexibility in geometry configurations in the SEL/HiFi code and is a milestone on the development path towards a more general multiblock implementation.
- Implemented in SEL a reacting plasma model which allows for two species -- singly ionized plasma and an energetic, dynamic neutral species. The model tracks mass, momentum, and energy exchange between species. Collisional and frictional effects are not yet included. Supported Dr. Nelson in conducting simulations that show momentum transfer from plasma to neutrals in an FRC-based propulsion application.
- Finalized overall mathematical model for three-component, interacting and reacting plasmas starting from the Boltzmann equation.
- The 3D equilibrium code is still under development. Improvements were made to the field line integrators and q calculators which were used to determine a lambda profile that is a function of the flux surfaces. This method of determining lambda has proven to be unworkable because it still finds a q value in ergodic regions where there are no surfaces and there is no simple way to clearly identify the separatrix. A new method has been developed that computes an artificial temperature function, T, which is a parallel diffusion steady state, satisfying div(bb*grad(T))=0 with T=1 on the magnetic axis and T=0 at the wall and b the magnetic field direction. Numerical solutions will always have a small perpendicular numerical diffusion to resolve the perpendicular dependence but it

must be small enough to allow the parallel diffusion to flatten the T profile in ergodic regions and islands. This is known to require high order elements which the code did not have but is being upgraded to include. When finished this upgrade will allow accurate computation of a T profile that is constant on a flux surface and flattened in ergodic regions and islands. The lambda profile may then be specified as any function of T.

- Assisted Chris Hansen in computing Taylor states for a proposed new helicity injector configuration for HIT-SI having 3 injectors on the same side driven with 3-phase AC power. Several different size entrance regions were used and the minimum ratio of toroidal spheromak flux to injector flux that produces closed flux surfaces was determined and found to decrease as the size of the entrance region increased.

Development of a Scalable Parallel Solver (*Alan Glasser*)

Accomplishments

Major progress has been made toward a scalable parallel solver, initially for the 2D SEL/HiFi code, but straightforwardly extendable to other 2D and 3D codes. After writing, testing, and debugging general-purpose solver routines to implement physics-based preconditioning, the remaining effort is devoted to developing and testing application-specific Schur complement routines for a sequence of increasingly realistic test problems. The first and simplest such problem is linear ideal MHD waves in a doubly periodic plane, with 7 dependent variables. Next, a nonlinear wave problem with 8 dependent variables has been developed and tested, initially with relative amplitude up to 10^{-2}. Beyond this amplitude, suppression of nonlinear coupling to short-wavelength modes requires the use of substantial dissipation. Incorporation of resistivity, viscosity, and thermal conductivity enables the testing to reach an amplitude of 10^{-1}. At this point, the motion is strongly nonlinear and dissipative.Scaling results up to 192 processors on the PSI Center ICE Cluster are shown in the accompanying log-log plot. Simplification of the operators used in physics-based preconditioning is found to accelerate convergence.Additional successful testing on this model, using homogeneous Dirichlet and Neumann boundary conditions for standing rather than traveling waves, has revealed required changes to the general formulation. After more scaling tests on this model, a Schur complement will be developed for a fully nonlinear model of a radially compressed FRC. Success in this endeavor will complete this phase of the program and should make it possible to extend the scalable solver to the 3D HiFi code.

Figure 1 is a log-log plot of wall clock seconds vs. number of processor on the PSI Center ICE cluster with 192 processors. There is a uniform background magnetic field in the x-y plane, and a plasma pressure with $\beta = 10\%$. The direction of wave propagation is 15^{o} from transverse. As the number of processors is scaled up, the size of the computational domain and the number of wavelengths in the domain are correspondingly increased, making this weak scaling. Each unit cell of the domain has one wavelength, 8 grid cells, and 6th degree polynomials in each of the two dimensions. The lowest, red curve is for a linear ideal MHD wave code with 7 dependent variables. The other curves are for a nonlinear, dissipative MHD code with 8 dependent variables, with wave amplitude relative to the uniform background ranging from 10^{-4} to 10^{-1}. The lowest 3 curves have no dissipation, while the top two have resistivity, viscosity, and thermal conductivity set to 10^{-2} and 10^{-1}, respectively, in order to suppress nonlinear coupling to unresolved short wavelength, high frequency modes. Each run consists of 64 time steps to one full wave period of the slow wave, implicitly stepping over the faster time scales of the fast and shear Alfven waves. The lowest 4 curves require only one Jacobian evaluation, while the highest one requires multiple Jacobian evaluations. Ideal weak scaling would approach a constant time to solution. The actual time to solution increases quite slowly with increasing size. The largest runs have 3,538,944 dependent variables and finish in about 6 minutes of wall time.

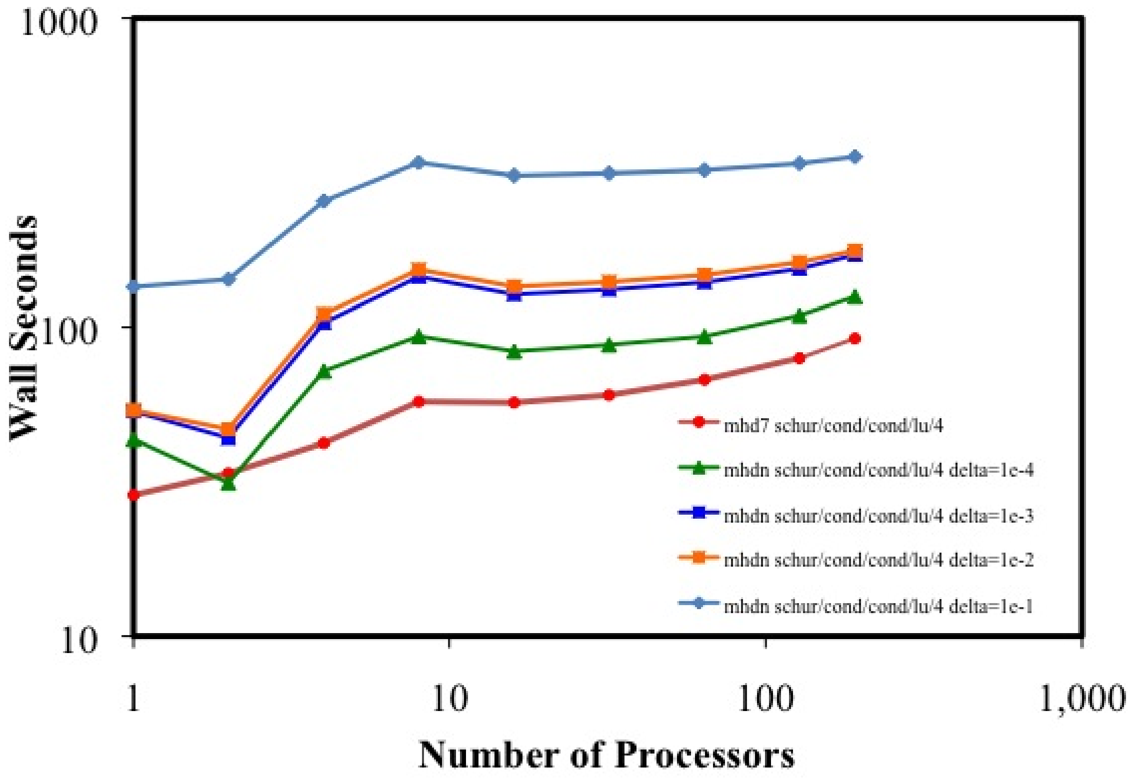


Figure 1. *Weak parallel scaling of physics-based preconditioning solver for nonlinear dissipative MHD waves in a doubly periodic plane.*

Two-fluid and Transport Group *(C. R. Sovinec, E. D. Held, J.-Y. Ji, and J. B. O'Bryan)*

Accomplishments

Over the quarter of funding ending on 9/30/09, the Two-fluid and Transport Group implemented the nonlinear Chodura model for anomalous resistivity in the NIMROD code and upgraded the stitch code to make it more general for assembling multiple regions of mesh. We have also made significant progress in developing closure relations for arbitrary collisionality in general magnetic geometry.

We have implemented the semi-empirical Chodura resistivity[1] to model conditions where electrons are accelerated beyond significant inertial and drag responses. Here, electron microinstabilities are important, and their macroscopic effects have been described by a semi-empirical resistivity formula:

$$\eta_{ch} = \frac{m_e \nu_{ch}}{ne^2}, \quad \nu_{ch} = C_c \omega_{pi} \left[1 - \exp\left(-f \left| {}^{v_d}\!/_{v_s} \right| \right) \right]$$

where v_d is the electron drift velocity and v_s is the acoustic speed. The NIMROD implementation is three-dimensional and implicit for the magnetic-field advance. The temperature and number-density dependencies in the resistivity are time-centered through NIMROD's temporal staggering during the advance of magnetic field. The computation of drift speed is based on current density at the start of each step, so computations using this implementation of Chodura resistivity are formally first-order accurate in time-step.

The stitch code is a utility that assembles adjacent, possibly multi-block, regions of mesh into a new mesh for NIMROD to provide greater geometric flexibility with relatively basic preprocessing for each region[2]. It does this by revising the data structure that defines the external boundary and by updating the data structures that define connections among grid blocks. The original implementation worked well for assembling pairs of regions where the stitch starts and ends on the new external boundary. We have generalized its operation to allow 'zippering' operations where only the start or end of a stitch is on the new domain boundary. This is important for assembling regions where more than two meet at a point, such as the case of a spherical domain shown in the figure below. Stitching regions 0 and 1 then regions 1 and 2 leaves a cut between regions 0 and 2. The new modifications allow a third stitch to remove the cut.

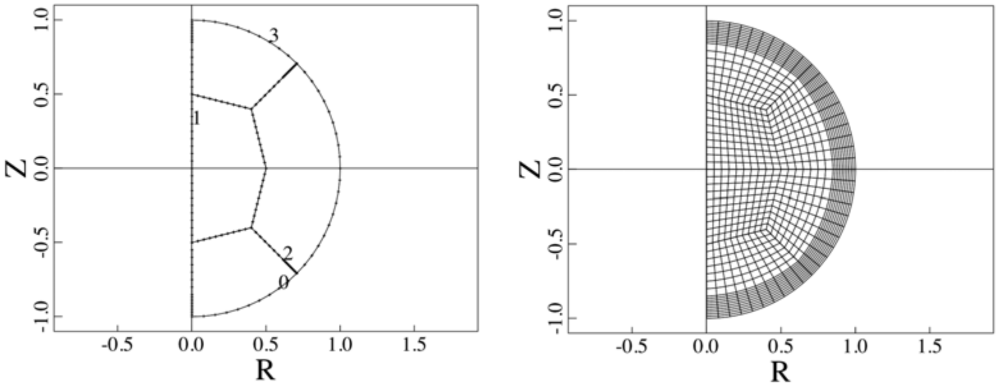


Figure 1. *Borders and labels of separate meshing regions (left) and final assembled mesh (right) for simulating the proposed spherical plasma dynamo experiment*[3] *at the Univ. of Wisconsin.*

There are also two items to note regarding analytical developments for closing fluid moment equations:

- We have used the general moment approach to formulate closure theory for general collisionality in general magnetic-field geometry. Applying the theory to toroidal plasmas, we have written explicitly the closure equations up to the second order of the small gyro-radius with no flux surface average.
- We have optimized the fitting functions that describe magnetization in transport coefficients for collisional plasma. Simplifying the polynomial in the denominator by dropping the $x^{4/3}$ term, i.e. replacing the denominator of our previous expression with

  $$x^{8/3} + a_4 x^{7/3} + a_3 x^{6/3} + a_2 x^{5/3} + a_1 x + a_0$$

  gives the same accuracy (<1%) in fitting.

---

[1] R. Chodura, "A hybrid particle-fluid model of ion heating in high Mach number shock waves", "*Nucl. Fusion* **15**, 65 (1975).

[2] See the Quarterly Progress Report of the PSI-Center, July-September, 2007.

[3] C. Forest, *et al.*, "MHD Modeling of a Plasma Dynamo Experiment," APS Bulletin Archives, BAPS.2008.DPP.UP6.35.

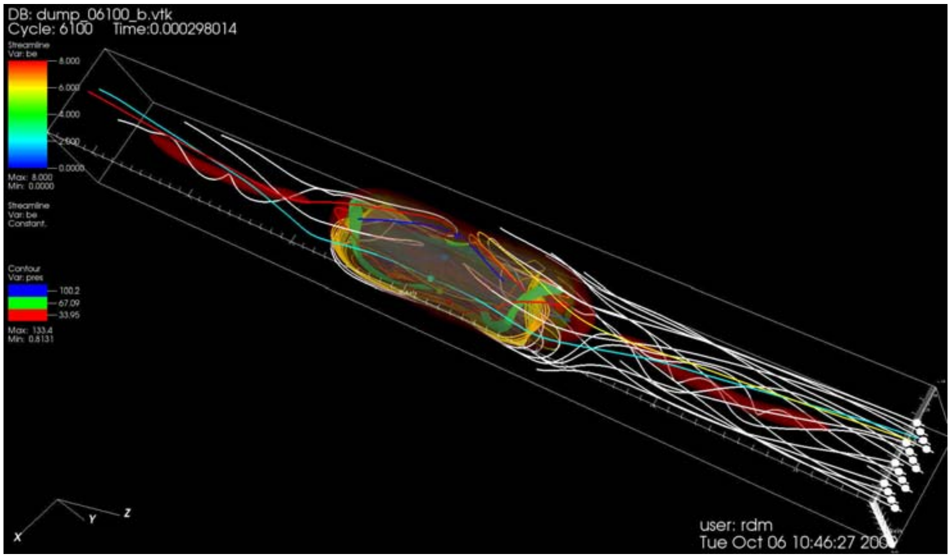
FLR and Kinetic Effects Group *(R. Milroy, C. Kim, and G. Cone)*

We continue with RFP stability studies and NIMROD simulations of rotating magnetic fields (RMF) to form and sustain FRCs.

Accomplishments

- Charlson spent a week in Madison collaborating with V. Mirnov and the MST group. NIMEQ has been added to NIMPSI that allows a Grad-Shafranov equilibrium to be solved on the NIMROD finite element grid. A modified version was used to generate MST like equilibria to study m=0 edge instabilities in RFPs.

- Good progress has been made in simulating the use of rotating magnetic fields (RMF) to form and sustain FRCs. Previously we reported that application of an end-shorting boundary condition led to a strong open field-line toroidal field resulting in a kink instability. We found that if we apply $B_\theta = 0$ on the radial boundary, corresponding to no net axial current, the toroidal field is reduced and the kink instability does not develop. Boundary conditions corresponding to odd-parity antennas have been developed and simulations show much longer field lines in the FRC, as predicted and shown in the figure below. A question remains as to whether the configuration could be sustained without opening field-lines. At this time we are still limited to running with relatively large resistivity, which limits the ratios of poloidal field to RMF field that we can achieve. Other accomplishments include simulations with shorter antennas to match the TCSU experiment, and the generalization of the antenna boundary conditions so that simulations with a shaped radial boundary can be performed. This will allow simulation of the concept of forming FRCs in a conical formation region and then translating them to another chamber. Simulations with lowered viscosity and with higher mode numbers (up to n=5) show MHD stability for the parameters that we have been operating at.



Pressure contours and magnetic field-lines for an odd-parity simulation. Colored lines originate on a rectangle at the axial midplane, while the white lines originate at one end of the simulation region.

Interfacing Group (*B. A. Nelson, C. C. Kim, S. D. Griffith*)

The IG is tasked with assisting in computational support for the twelve collaborating ICC experiments (along with the three physics groups).

Accomplishments:

- LDX simulations by J. Kesner continue. He is presently grappling with the interchange spectrum - the growth rate runs away with increasing toroidal mode number. The addition of gyroviscosity is being explored as a way of rolling off the spectrum.
- Coplanar coaxial flux injection simulations (the Caltech experiment) continue. Velocity boundary conditions were modified to allow normal inflow. Still no "fast kinking" *viz.*, before the flux reaches the far boundary, is observed. Simulations show large azimuthal flow. Is this flow the source of stability? If so, why is this flow absent in the experiment? Attempts at addressing these questions continue.
- NIMEQ has been added to NIMPSI that allows GS equilibrium to be solved on NIMROD finite element grid.
- Two-dimensional studies of a translating FRC interacting with background neutrals (through charge exchange, ionization, and recombination) in the HiFi (SEL) code are beginning:
  - An initial FRC equilibrium (20 eV, $n_e = 10^{19}$ m^{-3}, $B_{ext} = 0.1$ T) in 10^{19} m^{-3} peak density neutrals is compressed and translated by an external coil:

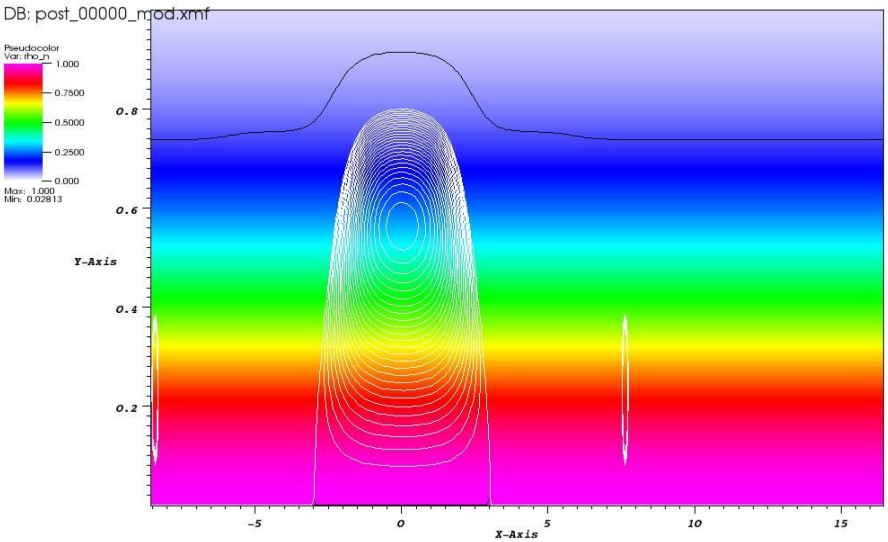


Figure IG-1

SEL simulation of a translated FRC (white flux contours) in dynamic background neutrals (pseudocolor): Initial equilibrium.

  - The high-density compressed FRC ionizes the neutrals near x=0, and builds momentum density, transferring momentum to neutrals through charge-exchange:

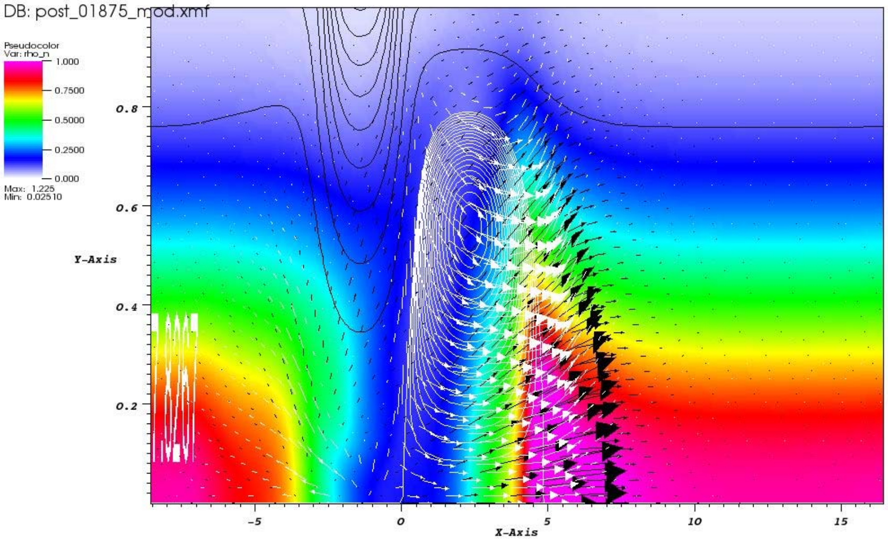


Figure IG-2

SEL simulation of a translated FRC (white flux contours, black contours from translation coil), dynamic neutrals (pseudocolor), ion momentum density (white arrows), and neutral momentum density (black arrows)

- The FRC is essentially stopped, having transferred its momentum to the neutrals through charge-exchange. A sound wave is traveling to the right..

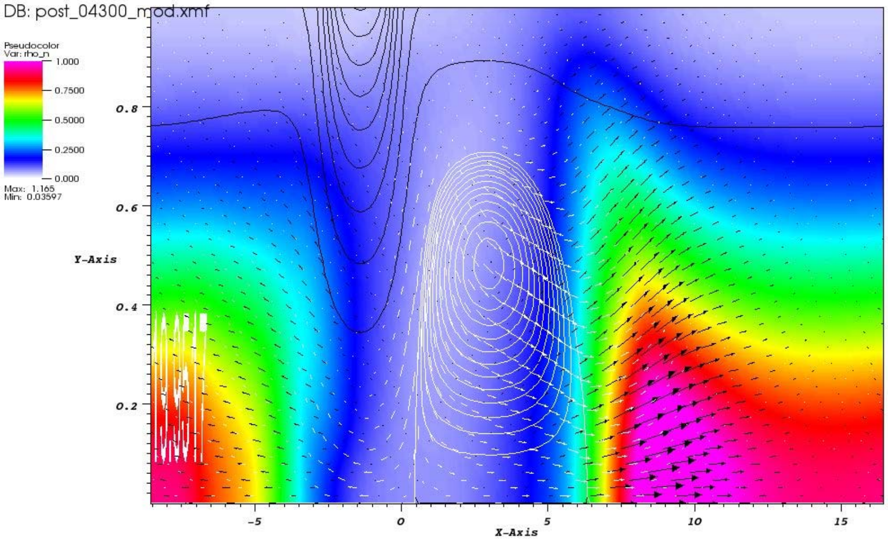


Figure IG-3

SEL simulation of a translated FRC (white flux contours, black contours from translation coil), dynamic background neutrals (pseudocolor), ion momentum density (white arrows), and neutral momentum density (black arrows).