Quarterly Progress Report of the PSI-Center (January - March 2009)

by

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The Plasma Science and Innovation Center (PSI-Center) has accomplished a great deal this quarter. The PSI-Center is organized into four groups: Boundary Conditions and Geometry, Two-Fluid and Transport, FLR and Kinetic Effects, and Interfacing. Each group has made good progress and the results from each group are given in detail.

Progress Report for the BC&G Group (*U. Shumlak, W. Lowrie, V.S. Lukin, G. Marklin, and E. Meier*)

Accomplishments

- Implemented a grid metric calculator in SEL for several metrics defined in two papers:
    - P.M. Knupp, Algebraic mesh quality metrics for unstructured initial meshes, Finite Element Anal. Design 39 (2003) 217-241
    - Y. Kallinderis and C. Kontzialis, A priori mesh quality estimation via direct relation between truncation error and mesh distortion, J. Comput. Phys. 228 (2009) 881-902
    - These metrics were calculated on grids with varying degrees of elementary distortions (stretch, shear, skew, and rotation).
- Implemented a parallel matrix eigenvalue solver using the SLEPc (Scalable Library for Eigenvalue Problem Computations) package.
    - The solver was used to find the extreme eigenvalues of the global spectral element stiffness matrix. The ratio of extreme eigenvalues (condition number) was then used as a grid metric.
- Investigated the effects of elementary grid distortions on solution quality in an effort to relate solution error magnitude with various grid metrics (ongoing)
    - Poisson's equation was used as a test problem to quantify errors associated with a second order operator.
    - The advection equation was used as a test problem for errors associated with a first order operator.
- Assisted Alan Glasser with merging in his branch of the SEL code that included physics-based preconditioning (PBP) with the main SVN code branch; similarly, assisted in the development of the complete code separation into an algorithm library libsel.a and a physics module;
- Transferred the algorithm/physics complete code separation technique to the HiFi code: the main algorithm is now compiled into a libhifi.a library that can be used by a free-standing physics module;
- Updated the existing documentation in the form of README files for both SEL and HiFi to reflect the above changes to the codes;

- Working with Alan Glasser, assisted in implementation and verification of the self-similarly contracting/expanding grid algorithm using cylindrical compression of an FRC configuration with the SEL code;
- Worked out various glitches in running SEL/HiFi on the new PSI-Center-shared ICE cluster and moved SVN repositories for both codes to ICE;
- Began development of the multi-block version of HiFi;
- Attended and presented the HiFi code with an invited mini-symposium talk at the SIAM Conference on Computational Science and Engineering (CSE09), in Miami, FL;
- Prepared for submission a paper on modeling highly anisotropic thermal diffusion with SEL. The paper explores the effects of overall spatial resolution, polynomial degree, and computational grid directionality on the accuracy of solutions using spectral element spatial discretization. These effects are qualitatively explained and numerically quantified.
- Implemented a non-reflecting boundary condition (for hyperbolic systems, and for MHD in particular) that allows for tangential flux gradients by placing a boundary condition on the normal derivatives of normal flux. Our previous approach was to specify the normal flux at the boundaries, but this was found to be reflective in the presence of tangential gradients.
- The PSI-TET code has been upgraded to include a set of diagnostic routines which interpolates magnetic fields to arbitrarily specified probe locations to facilitate comparisons with experimental data. Output files can now be written in TecPlot format as well as VTK. Data parallelization is still in progress. Domain decomposition with Metis has been installed and tested.
- A new 3D equilibrium code based on PSI-TET has been developed and is in its final testing phase. It solves J=Lambda*B with Lambda a non-linear function of B that is constant along the field lines. The first application will be to compute a 3D equilibrium for HIT-SI using a 2-Lambda model, where Lambda is assumed to have one constant value on open field lines that go through the injectors and a different constant value on closed field lines that remain inside the flux conserver. This is a model for a driven spheromak whose injector Lambda exceeds the average Lambda as typically occurs in the experiment and cannot be represented by a Taylor state.

Scalable Parallel Solvers (*Alan Glasser*)

Accomplishments

During the fourth quarter of 2008, the work completed was on developing a scalable parallel solver for extended MHD, using on physics-based preconditioning to reduce the size of the matrices to be solved and make them more diagonally dominant, and eithter FETI-DP or static condensation for scalable solution of the resulting matrices. These test were limited to ideal and Hall MHD waves in a doubly periodic plane. FETI-DP was found to be perfectly scalable, but limited to symmetric-positive-definite (SPD) matrices. Static condensation was found to be as efficient as FETI-DP up to 64 processors.

The main effort in the first quarter of 2009 has been extending these results to more interesting and realistic physics problems. The first task was to merge the version of the 2D HIFI code with the main line simultaneously developed by V. S. Lukin and students. This included moving from

PETSc Version 2.3.3 to Version 3.0.0, which resolved problems caused by known PETSc bugs. This also gave access to an improved interface to the Hypre/BoomerAMG algebra multigrid routines, which might be competitive for preconditioning. As part of this work, Alan reorganized the code to more cleanly separate the main solver routines from the physics routines, substantially simplifying the job of developing new physics models without modifying or recompiling the solver. A previously-developed FRC physics module has been modified to allow it to treat moving boundaries. The code has been ported to the new PSI Center parallel cluster and made to run very efficiently on it.

In developing scalable solver techniques, Alan has carefully separated the general-purpose code, incorporated into the solver modules, from the problem-specific routines, incorporated into the physics module. The main task of applying the methods is then to write a Schur complement routine for each physics problem. This work is now in progress for the FRC module.

Two-fluid and Transport Group *(C. R. Sovinec, E. D. Held, J.-Y. Ji and J. B. O'Bryan)*

Accomplishments

Over the quarter of funding ending on 3/31/09, the Two-fluid and Transport Group has made progress in issues related to nonlocal closure.

In our nonlocal analysis and computations, we determine closure information, such as heat flux, as a function of position by integrating kinetic contributions along characteristic paths. For example, the integral for parallel heat flux at position z_0 is

$$q_{\|}(z_0) = -\frac{1}{2} v_T T \int_{-\infty}^{+\infty} K(z_0 - z') \frac{n}{T} \frac{dT(z')}{dz'} dz' ,$$

where v_T is the thermal speed, and z is the path length along the characteristic normalized by the effective collisional mean-free-path length (v_T divided by the collision frequency). In the collisional limit, the kernel function K is essentially a Dirac delta-function, so the integral is proportional to the local gradient of temperature. For electrons, the characteristics are magnetic field trajectories that start from each point where the closure information is needed. While the kernel of each integral contains information about the plasma distribution function at each point, we have found that a numerical fitting can provide an accurate approximation for the integral heat flow closure. The fitting is

$$K(z) = -a(z) \ln\left(1 - e^{-|z|^{0.5466}}\right) ,$$

, and

$$a(z) = \begin{cases} 0.4260 + 0.7568 e^{-|z|^{0.1174}} & , \quad |z| \le 0.3040 \\ 0.7432 & , \quad |z| > 0.3040 \end{cases}$$

For conditions where the magnitude of the magnetic field varies weakly, the fitting is accurate to within 2% for any collisionality and reproduces parallel heat flows in the collisional (Braginskii) and collisionless limits. This is a surprising result. If the approximation can be generalized with respect to magnetic field variation, it will also make the nonlocal closure computations much more practical.

In replying to reviewers' comments on the manuscript "Summary of classical transport theories for electron-ion plasmas," that has been submitted to Physics of Plasmas, we clarified that a misunderstanding on the ion velocity to be order of ion thermal speed is due to misinterpretation of the perturbation expansion in the small collision length. We also verified the convergence of ion-electron transport coefficients with increasing numbers of moments as well as with increasing order of expansion in the small speed ratio of ions and electrons.

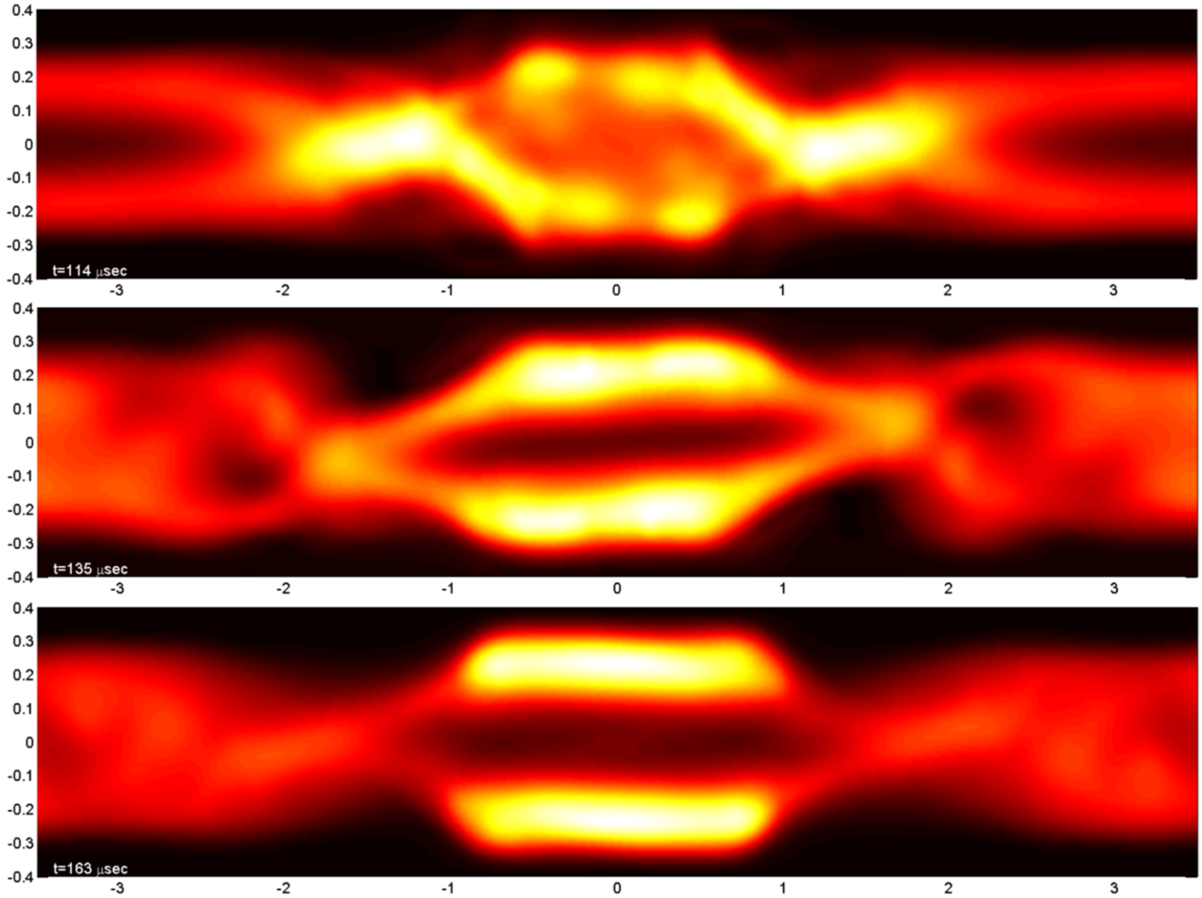
FLR and Kinetic Effects Group *(R. Milroy, C. Kim, and G. Cone)*

We continue with FRC simulations of formation, stability and RMF current drive.

Accomplishments

- NIMROD calculations to help the SSX team characterize the stability of an FRC formed by merging spheromaks continue. Results of a linear interchange instability survey demonstrate that the oblate FRC is stable to the interchange mode for separatrix beta (β_{sep} = ratio of max to min pressures) values greater than around 18%. The dependence of the linear n=1 interchange growth rate follows roughly the same trend with β_{sep} as previously found by E. Belova in her 2001 article "Numerical study of global stability of oblate field-reversed configurations".
- We have made good progress in simulating RMF current drive in an FRC using the NIMROD code. Numerically, this is a difficult problem where the Hall term is a zeroth order effect, and there is strong coupling between Fourier modes. We are now employing a two-fluid option in NIMROD with a finite electron mass set equal to 1/100 of the ion mass. Ideally we would like this ratio to be smaller, but the algorithm is more stable with this option. So far all of the calculations have been performed with only the n=0 and n=1 modes. We have made several variations of initial and boundary conditions:
  - Our first simulations were initialized with an existing FRC, and RMF boundary conditions that approximate an infinitely long antenna were applied. These simulations were not successful in sustaining the existing FRC which tended to decay away before the RMF penetrated sufficiently. A new FRC would begin to form but it tended to form in the end regions, rather than the location of the original FRC.
  - An attempt was then made to form an FRC from initial uniform cold plasma embedded in a forward bias magnetic field, with RMF boundary conditions that approximate an infinitely long antenna. The bias magnetic field had a 2× mirror at each end, to center the FRC when it formed. Again, reversal was achieved but it was not localized to the central region, so a well defined FRC could not be formed.
  - The RMF boundary conditions were then modified to approximate a finite length antenna. Initially this was done by simply specifying an n=1 E_z with an appropriate axial shape function to center it near the midplane. We then added an n=1 E_θ concentrated near the antenna ends to approximate the effects of the return straps on the TCSU antennas. With these boundary conditions, we have recently been successful in simulating the RMF formation of a well defined FRC from initial uniform cold plasma embedded in a forward bias magnetic field. These results are preliminary, but there are some interesting initial observations. The most significant is the fact that when magnetic field reversal begins, the plasma has a large n=1 distortion that looks like the

beginning of a strong tilt mode, but as the simulation proceeds the distortion becomes smaller leading to a symmetric FRC, as illustrated in the figure below. This simulation has a large viscosity, which reduces the growth rate for instabilities, but the fact that it becomes symmetric suggests the RMF may provide a restoring force. This will be investigated further. It is also noted that there is no evidence of an interchange instability or radial shift mode in these simulations. This study will continue.



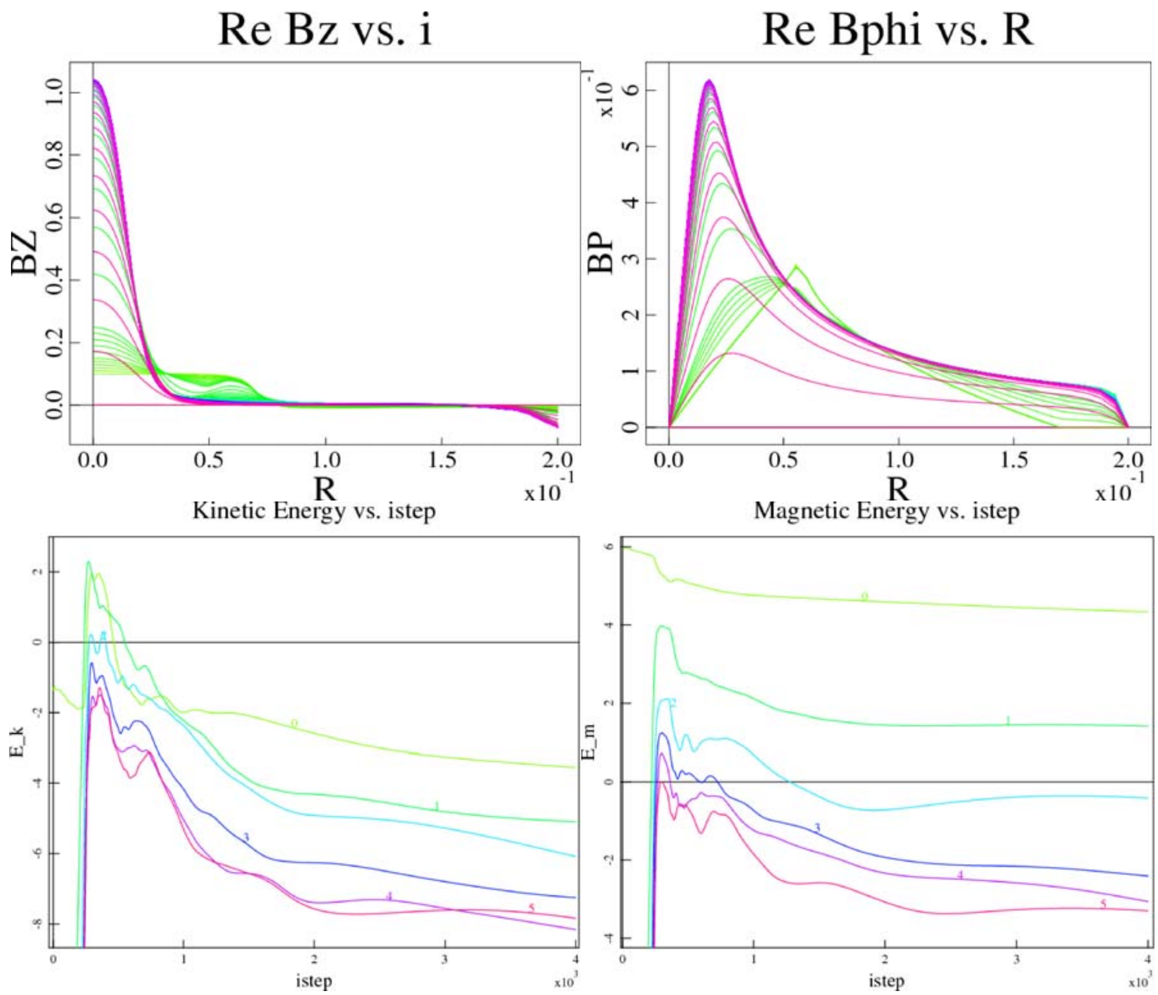
Pressure contours from a simulation of Rotating Magnetic Field formation of an FRC at three different times (114, 135, and 163 μsec). The figure shows a *tilt-like* distortion decay as the FRC forms.

Interfacing Group (*B. A. Nelson, C. C. Kim, S. D. Griffith*)

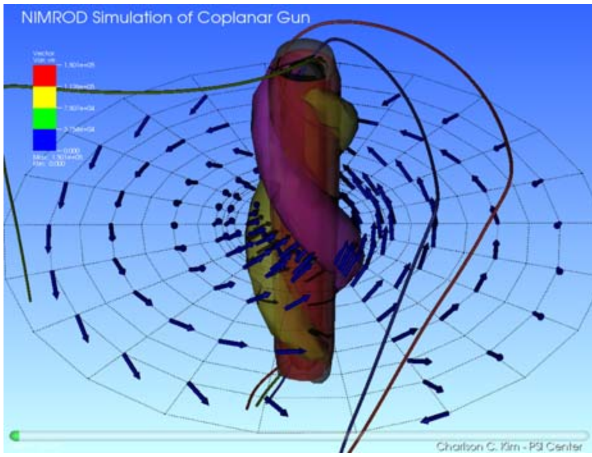
Accomplishments:

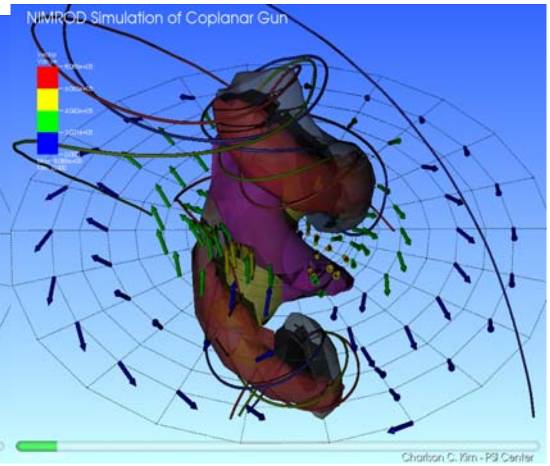
- The IG is tasked with assisting in computational support for the twelve collaborating ICC experiments (along with the three physics groups).
- The new cluster (purchased with an AFOSR-DURIP grant) is installed and has undergone preliminary implementation. It is a 192-processor SGI ICE Altix 8200 model, Xeon-based, 384 GB RAM, with a 6 TB RAID6 Infiniband storage node. This local facility is available to the PSI-Center, as well as other UW computational groups.
- Simulations are continuing for the Caltech experiment. We continue to explore nonlinear evolution of plasma column. This example simulation shows an initially unstable screw

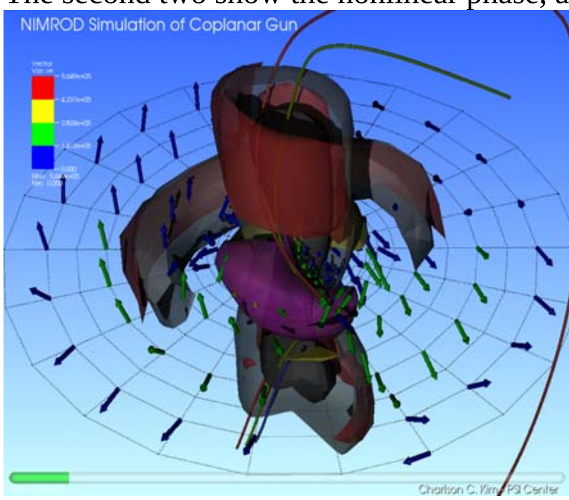
pinch evolve through the linear instability to nonlinear saturation. The initial screw pinch was generated with an axisymmetric simulation using a current injection model and an initial dipole field. The screw pinch profile shown below in the Bz and Bphi plots was allowed to evolve with axisymmetric mode numbers n=[0-5] modes. We show the kinetic energy and magnetic energy versus timestep. Amplitudes of the energy are on a logarithmic scale.

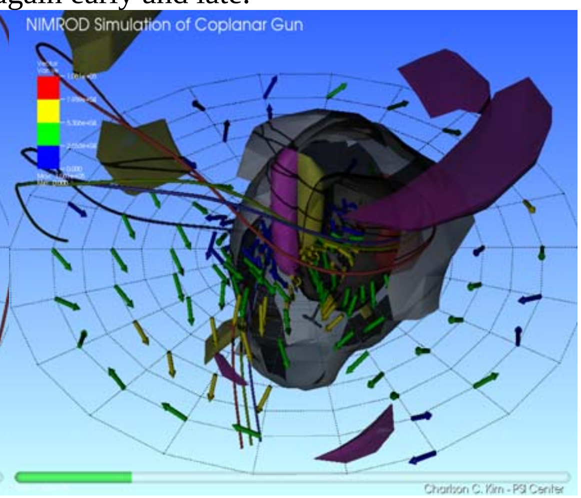


Shown in the sequence of visualizations are 90% Bphi (red contour), [60%,80%] |B| (light grey, dark grey/black), ±50% Vr(n=1) (purple,yellow), 3 colored magnetic field lines with fixed footpoints, midplane cut showing velocity (arrows) in the plane. The green bar is the time and can be correlated with the energy plots. These are resistive simulations with only B and V, no density evolution. Elecd=10, Pr=100. The simulation duration is $2x10^{-5}$s. The first two show the linear phase, early and late.





The second two show the nonlinear phase, again early and late.




The last two show the saturated settling phase, early and late.