

## Quarterly Progress Report of the PSI-Center (April - June 2009)

by

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The Plasma Science and Innovation Center (PSI-Center) has accomplished a great deal this quarter. The PSI-Center is organized into four groups: Boundary Conditions and Geometry, Two-Fluid and Transport, FLR and Kinetic Effects, and Interfacing. Each group has made good progress and the results from each group are given in detail.

### **Progress Report for the BC&G Group (*U. Shumlak, W. Lowrie, G. Marklin, and E. Meier*)**

#### **Accomplishments**

- Continued investigating the effects of grid distortion on solution quality using Poisson's equation and the advection equation as test problems (Laplacian and Grad Operators) (ongoing)
- Analyzed five different types of distortions for each test problem: stretch, shear, skew, large angle, and small edge distortions.
- A significant finding was that even with large grid distortions, the spatial convergence rates for the high-order (spectral) element method remained. The error increased by some scalar function with increasing grid distortion. This was seen by plotting the L2 norm for various resolutions and degrees of distortion for each test problem and each grid deformation type.
- The product of the Knupp metrics (P.M. Knupp, Algebraic mesh quality metrics for unstructured initial meshes, Finite Element Anal. Design 39 (2003) 217-241) as well as the spectral element stiffness matrix condition number were found to provide good indicators of solution error.
- Submitted a paper to Computer Physics Communications and received comments. The paper describes modeling highly anisotropic thermal diffusion with SEL. Revision of this paper is in progress.
- For hyperbolic problems including the Euler equations and ideal MHD, a non-reflecting boundary condition (NRBC) has been implemented. Simulation experience shows that this NRBC is robust for problems with smooth interior solutions. When dissipation is added to the ideal MHD system, instabilities are seen when imposing the NRBC which was based on ideal MHD. (Meanwhile, the Euler system with dissipation behaves well when imposing the NRBC based on ideal Euler.) Work is under way to develop an NRBC for dissipative MHD with attention to well-posedness and stability.
- The 3D equilibrium code is still under development. It solves  $J = \Lambda * B$  on a tetrahedral mesh with  $\Lambda$  a non-linear function of  $B$  that is constant along the field lines. The code computes  $B$  for a given  $\Lambda(x,y,z)$  (this part is working) and then iterates to make  $\Lambda$  constant along  $B$ . Initial runs tried a 2- $\Lambda$  model, where  $\Lambda$  was assumed to have one constant value on open field lines that go through the injectors and a different constant value on closed field lines that remain inside the flux

consolver. This model did not converge, possibly due to errors associated with trying to represent a discontinuous current profile on an unstructured mesh. The field line tracers are being upgraded to construct a continuous representation of the flux surfaces, where they exist, and to define a continuous Lambda model which should have better convergence.

- A 2D Grad-Shafranov equilibrium solver has been implemented using COMSOL Multiphysics. It has been used to compute effective length coefficients that can predict a toroidal current from surface magnetic field measurements by probes or flux loops. It will also be used to attempt experimental equilibrium reconstruction by fitting surface measurements to a computed linear lambda model equilibrium. Work has begun on a companion 2D stability solver which will compute ideal, resistive and Hall MHD growth rates and eigenmodes for any equilibrium.
- Derived MHD equations for three-component, interacting and reacting plasmas starting from the Boltzmann equation. Electron, ion, and neutrals are the three components.

### **Scalable Parallel Solvers (*Alan Glasser*)**

#### **Accomplishments**

- Alan participating in a trip to Washington DC to participate in our proposal defense, April 6-7; and the Sherwood Meeting in Denver May 1-5, where I gave an invited CEMM presentation and a poster session on his solver developments.
- He put in a fair amount of effort with Eric Meier on his open boundary conditions, helping with literature searches, extended discussions, writing a small code for proof of principle, and development of a new strategy which appears very promising to solve that problem.
- With Brian Nelson, Alan visited John Slough and discuss his ELF project. We agreed that it would be preferable to separate 2D neutral entrainment modeling from 3D RMF formation and do it with the SEL code. This can be done as an enhancement to my existing FRC model, adding neutrals and translation. He has written a code model implementing all of Braginskii's transport model, which is applicable to the regime of operation of ELF.

### **Two-fluid and Transport Group (*C. R. Sovinec, E. D. Held, J.-Y. Ji and J. B. O'Bryan*)**

#### **Accomplishments**

Over the quarter of funding ending on 6/30/09, the Two-fluid and Transport Group has used information from general-moment computations to produce closure information for low-order systems. The group has also begun to implement the collisional magnetization effect for 3D variations in collisionality.

Regarding general-moment computations, we have made fitting formulas for two of the electron vector moments (heat flow and friction). The results provide better accuracy in the collisional limit than Braginskii's closures, which are based on using two moments. Some of the Braginskii coefficients are in error by as much as 65% for some finite values of  $x$  and have significant error for the large- $x$  limit, where  $x \equiv \Omega\tau$ ,  $\Omega$  is the gyrofrequency, and  $\tau$  is the collision time for a

given species. The new results from fitting are based on calculations with 160 moments for  $x < 100$  and on the asymptotic form for large  $x$ -values. The new formulas are practically exact with less than 1% error for the whole range of  $x$  and  $Z$  (the ion charge, checked up to  $Z=100$ ). We also have written two papers on the general moment method that address the following topics:

- (1) Analytical solution of the kinetic equation for a uniform single-component plasma in a magnetic field, and
- (2) Exact full Coulomb collision operators in the moment expansion. Here, we have written explicit formulas to calculate nonlinear terms of Landau-Fokker-Planck operators for arbitrary mass and temperature ratios.

We have begun to extend the NIMROD implementation of magnetization in collisional Braginskii closure relations to allow 3D variation of  $x$ . For example, the perpendicular thermal conductivity is

$$\kappa_{\perp} = \kappa_0 \frac{nT\tau}{m} \frac{(\gamma_1 x^2 + \gamma_0)}{(x^4 + \delta_1 x^2 + \delta_0)}$$

for each species, where  $\kappa_0$ ,  $\gamma_0$ ,  $\gamma_1$ ,  $\delta_0$ , and  $\delta_1$  are numerical constants. Like other 3D thermal dependencies, the 3D variations in magnetization are computed as functions of the periodic coordinate in NIMROD. They lead to algebraic coupling among Fourier components during the advance of temperature, and their average over the periodic coordinate is used during preconditioning operations when solving the respective algebraic system. A corrector step is used to make the advance second-order accurate with nonlinear thermal conductivities.

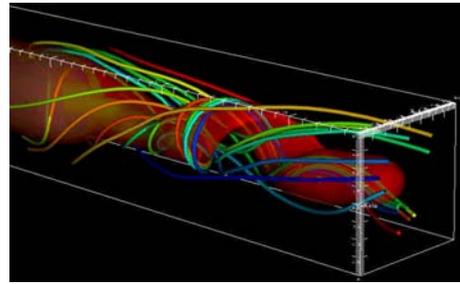
### **FLR and Kinetic Effects Group (*R. Milroy, C. Kim, and G. Cone*)**

We continue with improvements to the particle routines, RMF current drive, and FRC stability calculations.

#### **Accomplishments**

- Improvements to the particle subroutines have been incorporated in NIMPSI. The parallelization has been improved to work with general rblock decomposition. Further work is needed for decomposition in nlayers. The necessary algorithmic improvements have been formulated but yet to be implemented.
- The particle subroutines have been improved and reorganized to minimize the coupling to the main NIMROD code.
- A new particle field evaluator is being implemented. This should reduce the amount of necessary communication between time steps and improve performance.
- Work continues on high order finite element particle methods.
- We continue to communicate with the MST group, particularly Vladimir Mirnov, and have arranged a visit in late July/early August to continue collaboration on studying the impact of FLR effects on RFP devices.

- We continue work on simulating Rotating Magnetic Field (RMF) current drive to form and sustain an FRC using the NIMROD code. During this quarter work has focused on improved boundary conditions, both to represent a finite length antenna, and to include the effects of end-shortening at the axial boundaries. Previous calculations assumed axial periodicity. Preliminary results indicate that the end-shortening boundary conditions can lead to the generation of significant toroidal field in the open field line region between the FRC and the end walls. This field seems largest during the early formation phase and can lead to kink instability in this region, as shown in the figure. Work on refining these boundary conditions and the investigation of RMF physics will continue into the next quarter.



Pressure contours and magnetic field lines in the open field line region between the FRC (on the left) and the wall (on the right).

- Nimrod calculations to study of the tilt and interchange instabilities for a wall supported oblate FRC in the SSX parameter range is continuing. These instabilities are being compared with those of a conventional FRC with a cylindrical flux-conserver and with open field line flux separating the FRC from the wall. The scaling of growth rate with resistivity and viscosity is also being studied. In the next quarter we will begin studies of FRC formation by merging opposite helicity spheromaks.

### **Interfacing Group (B. A. Nelson, C. C. Kim, S. D. Griffith)**

#### **Accomplishments:**

- The IG is tasked with assisting in computational support for the twelve collaborating ICC experiments (along with the three physics groups).
- The new SGI “ICE” cluster (purchased with an AFOSR-DURIP grant) is now under routine operation by the PSI-Center and other local computational groups. All major PSI-Center codes have been ported, compiled, and are running. It is proving to be very useful for PSI-Center code development.
- Simulations of coplanar flux injection (similar to Caltech experiment, see Figure IG-1) and spheromak formation continue. Reproduction of even qualitative features of the experiment remains elusive with a resistive MHD model. The primary shortcoming seems to be that the plasma is ejected too fast from the gun region. It is unclear whether this is a shortcoming of the resistive MHD model or inadequate implementation of the boundary conditions that mimic the current injection.
  - For these resistive MHD simulations, the ejected plasma forms a stable screw pinch that fills the length of the domain. Once the column spans the entire length, the axial flow stagnates and the pinch contracts to a kink unstable configuration (Figures IG-2,3). We have followed the evolution of the kink instability to the ensuing formation of a spheromak (Figure IG-4). We reproduce many of the features observed by C. Sovinec *et al.*, PoP 2001 vol 8, in earlier simulations

using a different flux injection model. These preliminary simulations show the formation of closed flux surfaces and magnetic islands (Figure IG-5).

- We continue to study the boundary conditions and will look for areas of improvements to the model. We will also step ahead to two-fluid simulations of the coplanar gun. It is conjectured that the two-fluid model will have much smaller flows associated with the plasma ejection.
- We continue to support NIMROD simulation of LDX by Jay Kesner. Initial simulations have run into resolutions limits. Simulations with higher resolution are continuing.
- We continue to improve the NIMPSI branch of NIMROD and its administration. It is now hosted by an external HTTP-based service for improved access to collaborators, off-site backups, and administration off-loading.

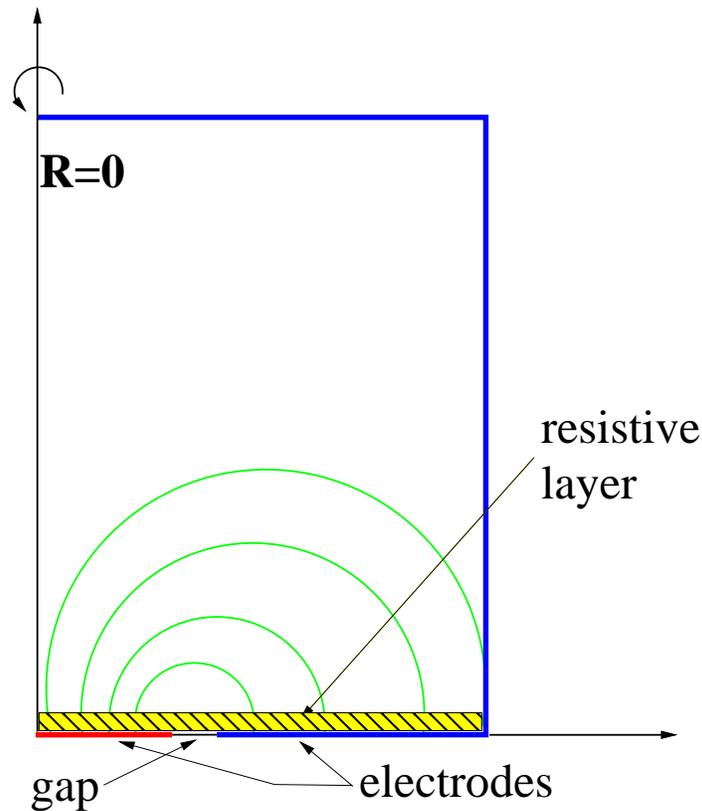


Figure IG-1

*Schematic of coplanar simulation geometry. Resistive layer thickness is exaggerated for illustration.*

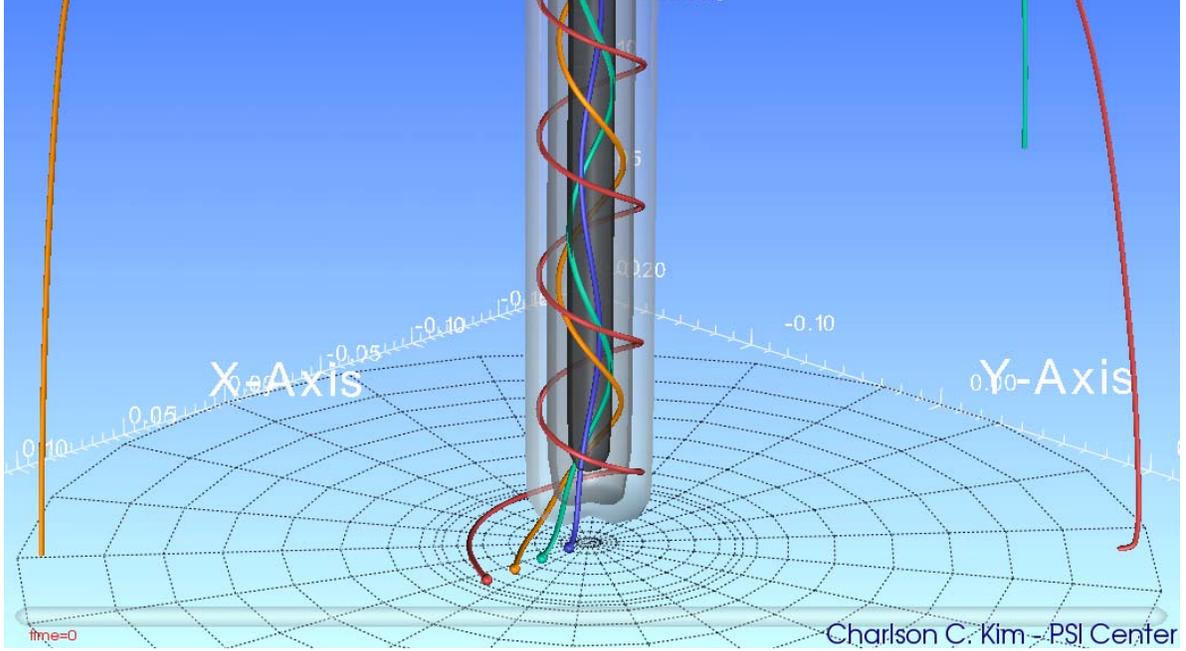


Figure IG-2

Magnetic field lines and  $|B|$  contours (in grey) show initial plasma column spanning the domain.

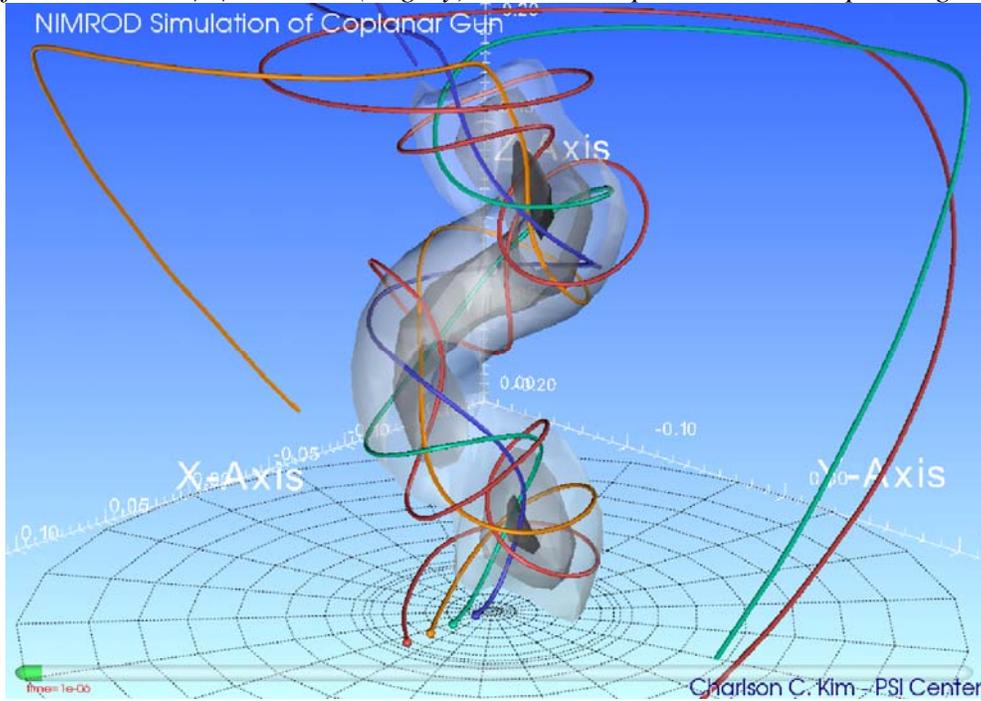


Figure IG-3

Late linear phase of kink instability.

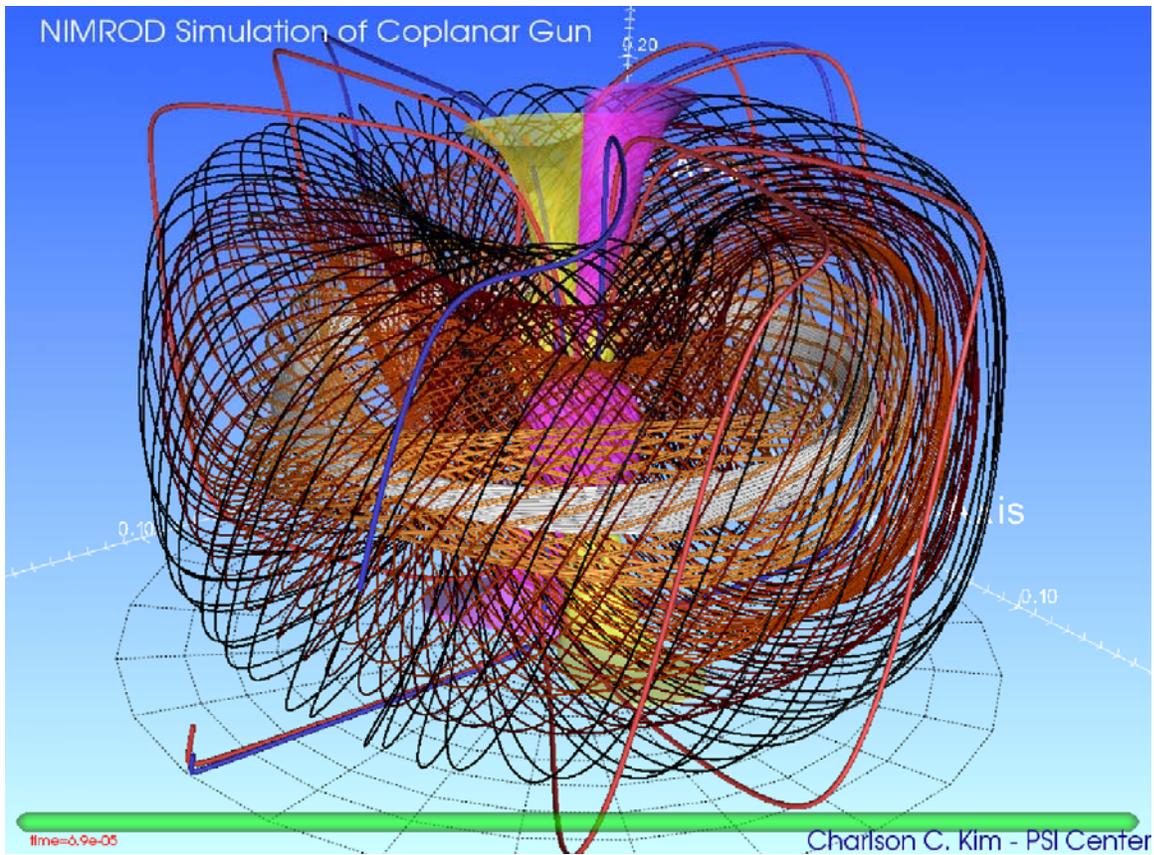


Figure IG-4  
*Magnetic field line visualization shows nesting of spheromak configuration.*

# Surface of Section

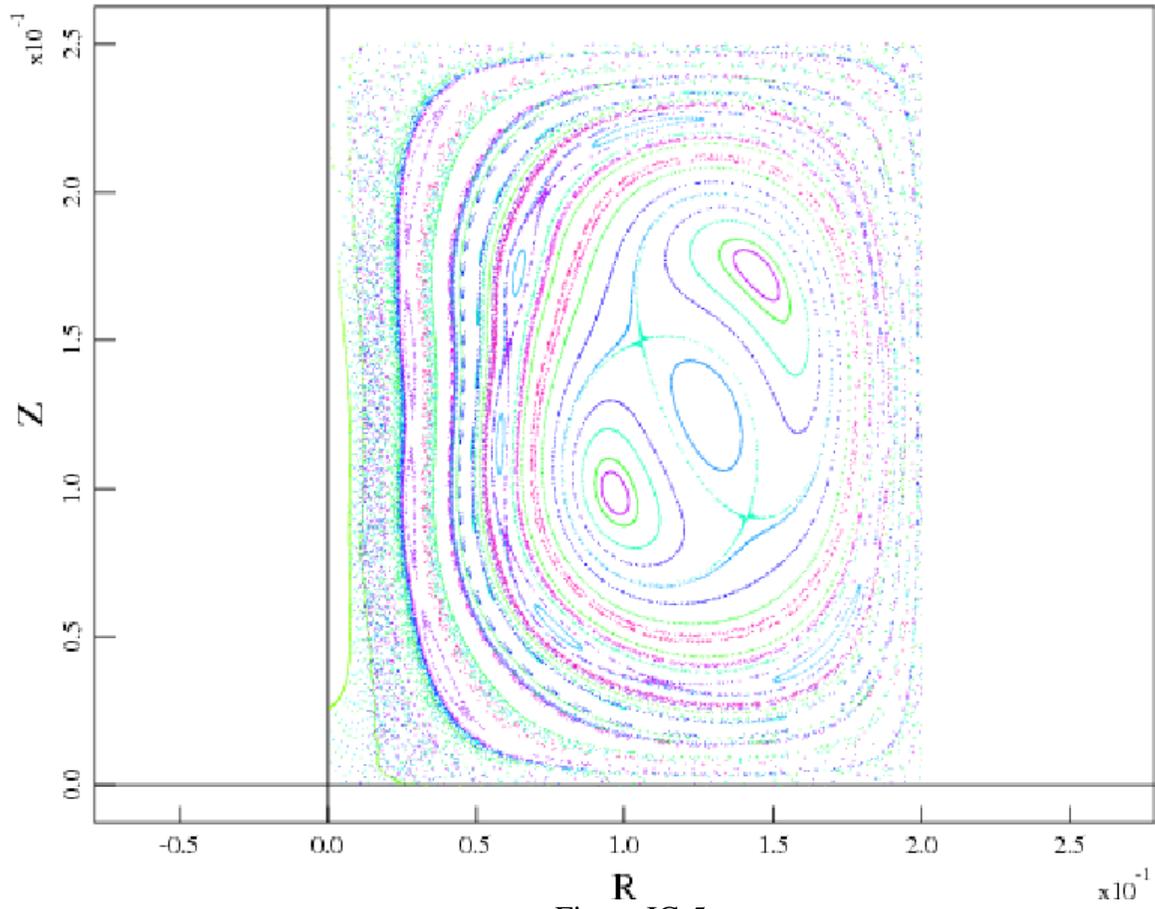


Figure IG-5

*Poincaré surface of sections show rich topology of surfaces and islands.*