# Progress and Goals for Transport: Closures for Partially Ionized Plasmas and $\cdots$

Jeong-Young Ji/Transport Kinetic Effects Group

Utah State University

PSI-Center Meeting

July 26, 2018, Seattle, WA

# Overview

- Fluid equations and closures
  - ◇ General moment equation approach
- Work accomplished (last 13 years)
- Work in progress
  - ◇ Parallel moment equations in NIMROD (Hankyu Lee)
  - ◇ Parallel closures in an inhomogeneous magnetic field
    - ⋆ Arbitrary collisionality
  - ◇ Closures for partially ionized plasmas
- Future work
  - ◇ To be completed in the near future
  - ◇ Possible work

# Fluid equations and closures/transport

Maxwellian moment $(n_a, \mathbf{V}_a, T_a)$ equations

(0,0) $d_t n_a + n_a \nabla\cdot\mathbf{V}_a = 0\,(d_t \equiv \partial_t + \mathbf{V}_a\cdot\nabla)$

(0,1) $\frac{3}{2} n_a d_t T_a + n_a T_a \nabla\cdot\mathbf{V}_a + \nabla\cdot\mathbf{h}_a + \nabla\mathbf{V}_a : \boldsymbol{\pi}_a = Q_a$

(1,0) $m_a n_a d_t \mathbf{V}_a - n_a q_a(\mathbf{E} + \mathbf{V}_a\times\mathbf{B}) + \nabla p_a + \nabla\cdot\boldsymbol{\pi}_a = \mathbf{R}_a$

General moment equations $D\mathsf{n} + \Omega\mathbf{b}\breve{\times}\mathsf{n} = C\mathsf{n}$ ($\mathsf{n}^{lk} \to v^{l+2k}$ moment)

$$(1,1)\ d_t\mathbf{h} + \Omega\mathbf{b}\times\mathbf{h} + \frac{7}{5}(\nabla\cdot\mathbf{V})\mathbf{h} + \frac{7}{5}\mathbf{h}\cdot(\nabla\mathbf{V}) + \frac{2}{5}(\nabla\mathbf{V})\cdot\mathbf{h} + \frac{5p}{2m}\nabla T$$
$$+\frac{T}{m}\nabla\cdot\boldsymbol{\pi} + \frac{7}{2}\frac{\nabla T}{m}\cdot\boldsymbol{\pi} - \mathbf{a}\cdot\boldsymbol{\pi} + \nabla\cdot\boldsymbol{\theta} + \frac{1}{3}\nabla\mathsf{u}^{02} + \nabla\mathbf{V} : \mathsf{u}^{30}$$
$$= C^1_{10}\mathbf{V}_{\mathrm{ei}} + C^1_{11}\mathbf{h} + C^1_{12}\mathbf{r} + \cdots\ (\mathbf{h}\text{ heat flow})$$

$$(1,2)\ d_t\mathbf{r} + \Omega\mathbf{b}\times\mathbf{r} + \cdots = C^1_{10}\mathbf{V}_{\mathrm{ei}} + C^1_{21}\mathbf{h} + C^1_{22}\mathbf{r} + \cdots\quad (\mathbf{r}\text{ heat heat flow})$$

$$(2,0)\ d_t\boldsymbol{\pi} + \Omega\mathbf{b}\breve{\times}\boldsymbol{\pi} + (\nabla\cdot\mathbf{V})\boldsymbol{\pi} + 2\overline{\boldsymbol{\pi}\cdot(\nabla\mathbf{V})} + p\mathsf{W} + \frac{4}{5}\overline{\nabla\mathbf{h}} + \nabla\cdot\mathsf{u}^{30}$$
$$= C^2_{00}\boldsymbol{\pi} + C^2_{01}\boldsymbol{\theta} + \cdots\ (\boldsymbol{\pi}\text{ viscosity})$$

$$(2,1)\ d_t\boldsymbol{\theta} + \Omega\mathbf{b}\breve{\times}\boldsymbol{\pi} + \cdots = C^2_{10}\boldsymbol{\pi} + C^2_{11}\boldsymbol{\theta} + \cdots\ (\boldsymbol{\theta}\text{ heat viscosity})$$

where $\mathbf{a} = \frac{q}{m}(\mathbf{E} + \mathbf{V}\times\mathbf{B}) - d_t\mathbf{V}$ and $\mathsf{W} = \nabla\mathbf{V} + (\nabla\mathbf{V})^{\mathrm{T}} - \frac{2}{3}\nabla\cdot\mathbf{V}\mathsf{I}$

Closures: express $\mathbf{h}_a(\mathsf{n}^{11}_a)$, $\boldsymbol{\pi}_a(\mathsf{n}^{20}_a)$, $Q_a$, $\mathbf{R}_a$ in terms of $n_a, \mathbf{V}_a, T_a$

Electron closures for high collisionality (Braginskii)

$\mathbf{h}_{\mathrm{e}} = (\beta)(\mathbf{V}_{\mathrm{ei}}) - (\kappa)(\nabla T_{\mathrm{e}})$, $\mathbf{R}_{\mathrm{e}} = -\mathbf{R}_{\mathrm{i}} = -(\alpha)(\mathbf{V}_{\mathrm{ei}}) - (\beta)(\nabla T_{\mathrm{e}})$

Transport: relate flux densities $\mathbf{h}_{\mathrm{e}}$, $\mathbf{J}$ to thermodynamic forces $\nabla T_{\mathrm{e}}$ and $\mathbf{E}$

$\mathbf{h}_{\mathrm{e}} = (\tilde{\alpha})\mathbf{E} - (\tilde{\kappa})(\nabla T_{\mathrm{e}})$, $\mathbf{J} = (\tilde{\sigma})\mathbf{E} - (\tilde{\alpha})(\nabla T_{\mathrm{e}})$

# Work accomplished - PSI Center and CEMM

- General moment equations: exact calculation of collision operators
  - ◇ Linear and nonlinear terms in total- and random-velocity moment expansions
    - ⋆ "Exact linearized Coulomb collision operator in the moment expansion", Phys. Plasmas **13**, 102103 (2006).
    - ⋆ "Landau collision operators and general moment equations for an electron-ion plasma", Phys. Plasmas **15**, 102101(2008).
    - ⋆ "Full Coulomb collision operator in the moment expansion", Phys. Plasmas **16**, 102108 (2009).
    - ⋆ "Analytical solution of the kinetic equation for a uniform plasma in a magnetic field", Phys. Rev. E **82**, 016401 (2010).
    - ⋆ "A framework for moment equations for magnetized plasmas", Phys. Plasmas **21**, 042102 (2014).
- Closures and transport for high collisionality
  - ◇ Large $x = \Omega\tau$ correction for electrons
  - ◇ Effects of ion-electron collisions on ion transport
    - ⋆ "Closure and transport theory for high-collisionality electron-ion plasmas", Phys. Plasmas **20**, 042114 (2013).
    - ⋆ "Ion closure theory for high collisionality revisited", Phys. Plasmas **22**, 062114 (2015).

# Work accomplished - PSI Center and CEMM (cont.)

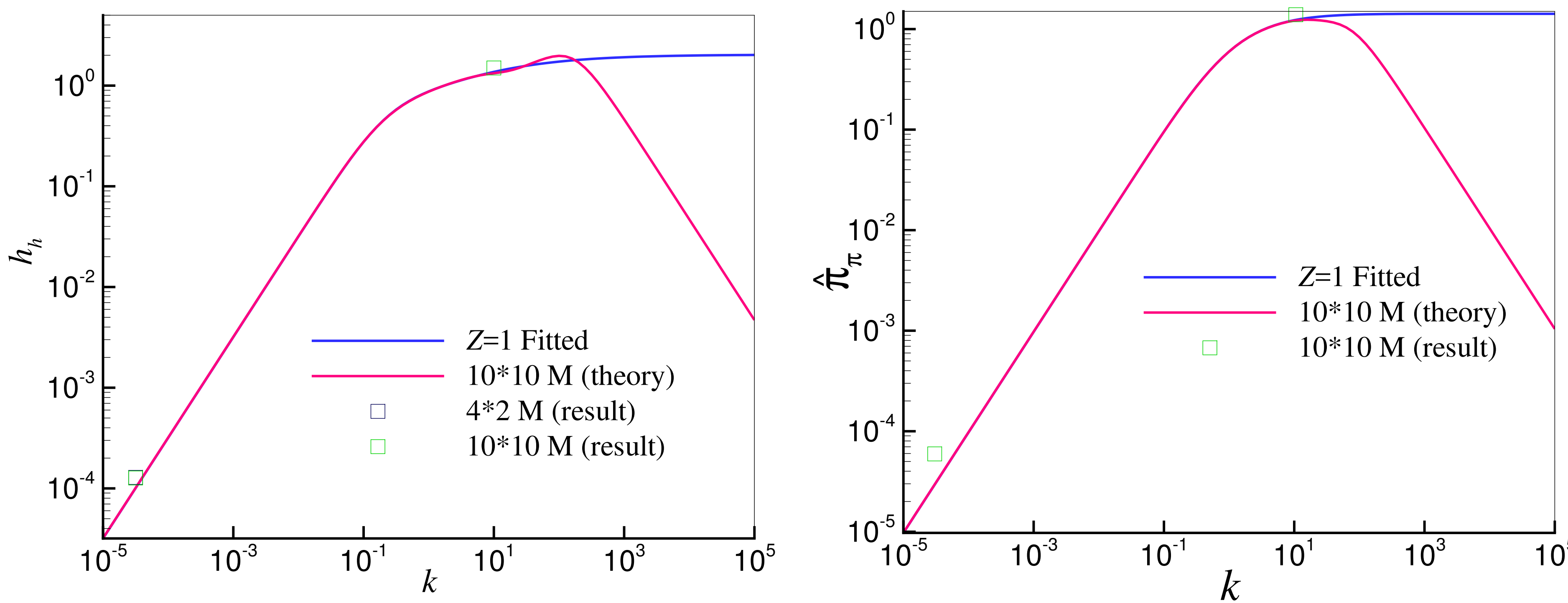
- Parallel closures and transport for arbitrary collisionality
  - ◇ Electron: for ion charge number $1 \leq Z_{\mathrm{eff}} \leq 10$
  - ◇ Ion: for various $AZ^2$ $(A = m_{\mathrm{i}}/m_{\mathrm{p}})$ and $T_{\mathrm{i}}/T_{\mathrm{e}} \leq 10$
    - ⋆ "Moment approach to deriving parallel heat flow for general collisionality", Phys. Plasmas **16**, 022312 (2009).
    - ⋆ "Moment approach to deriving a unified parallel viscous stress in magnetized plasmas", J. Fusion Energy **28**, 170 (2009).
    - ⋆ "Linearly exact parallel closures for slab geometry", Phys. Plasmas **20**, 082121 (2013).
    - ⋆ "Electron parallel closures for arbitrary collisionality", Phys. Plasmas **21**, 122115 (2014).
    - ⋆ "Electron parallel closures for various ion charge numbers", Phys. Plasmas **23**, 032124 (2016).
    - ⋆ "Electron heat flow due to magnetic field fluctuations", Plasma Phys. Control. Fusion **58**, 042001 (2016).
    - ⋆ "Ion parallel closures", Phys. Plasmas **24**, 022127 (2017).

# Work accomplished - PSI Center and CEMM (cont.)

- Since the last annual PSIC meeting
  - ◇ "Electron parallel transport for arbitrary collisionality", Phys. Plasmas **2**4, 112121 (2017).
  - ◇ "Electron parallel closures for the 3+1$(n, V_{\|}, p_{\|}, p_{\perp})$ fluid model", Phys. Plasmas **25**, 032117 (2018).
  - ◇ Electron: for ion charge number $Z_{\mathrm{eff}} > 10$ (Hankyu Lee)
    - ⋆ Paper in preparation
  - ◇ Parallel moment equations implemented in NIMROD (Hankyu Lee)
    - ⋆ Being verified against the theoretical integral closures
    - ⋆ Nonlinear terms being implemented
  - ◇ Parallel closures in an inhomogeneous magnetic field developed
    - ⋆ Paper in preparation
    - ⋆ Bootstrap current to be calculated
  - ◇ Closures for partially ionized plasmas

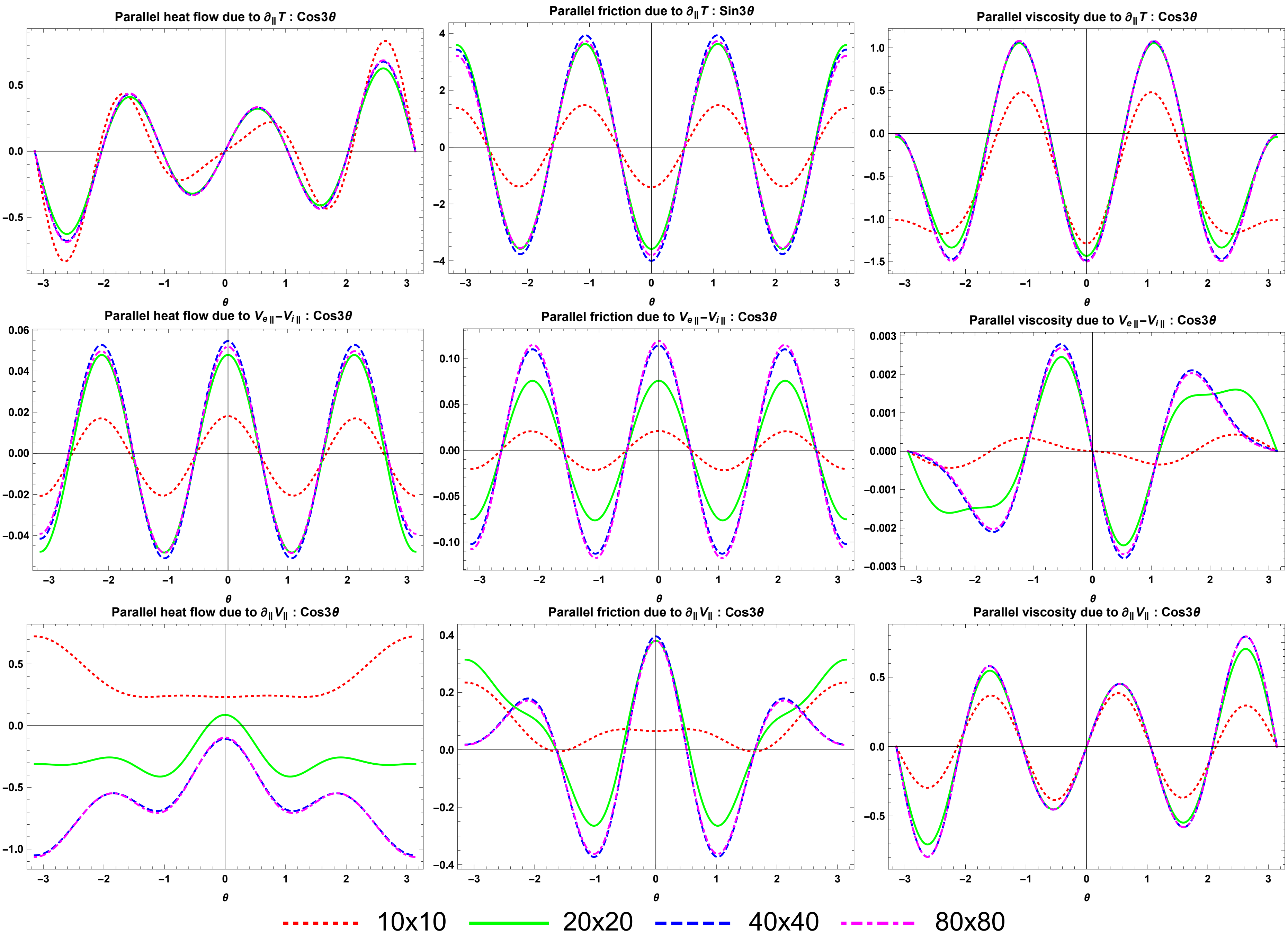
# Parallel moment equations in NIMROD

$$\frac{\partial}{\partial t}[n]+v_T\Psi\partial_\parallel[n]+\underbrace{v_T(\partial_\parallel \ln B)\Psi_B[n]}_{\text{inhomo. magectic field}}+\underbrace{v_T(\partial_\parallel \ln T)\Phi[n]+\frac{q}{2T}E_\parallel\Theta[n]}_{\text{nonlinear coupling terms}}=\frac{1}{\tau}c[n]+[g]$$




Parallel heat flow and viscosity responding to the temperature and flow velocity gradients, respectively. The NIMROD results (squares) are compared to the analytical integral closures.

# Parallel closures in an inhomogeneous magnetic field
$(\epsilon = r/R = 0.5)$



# Kinetic equations for a partially ionized plasma

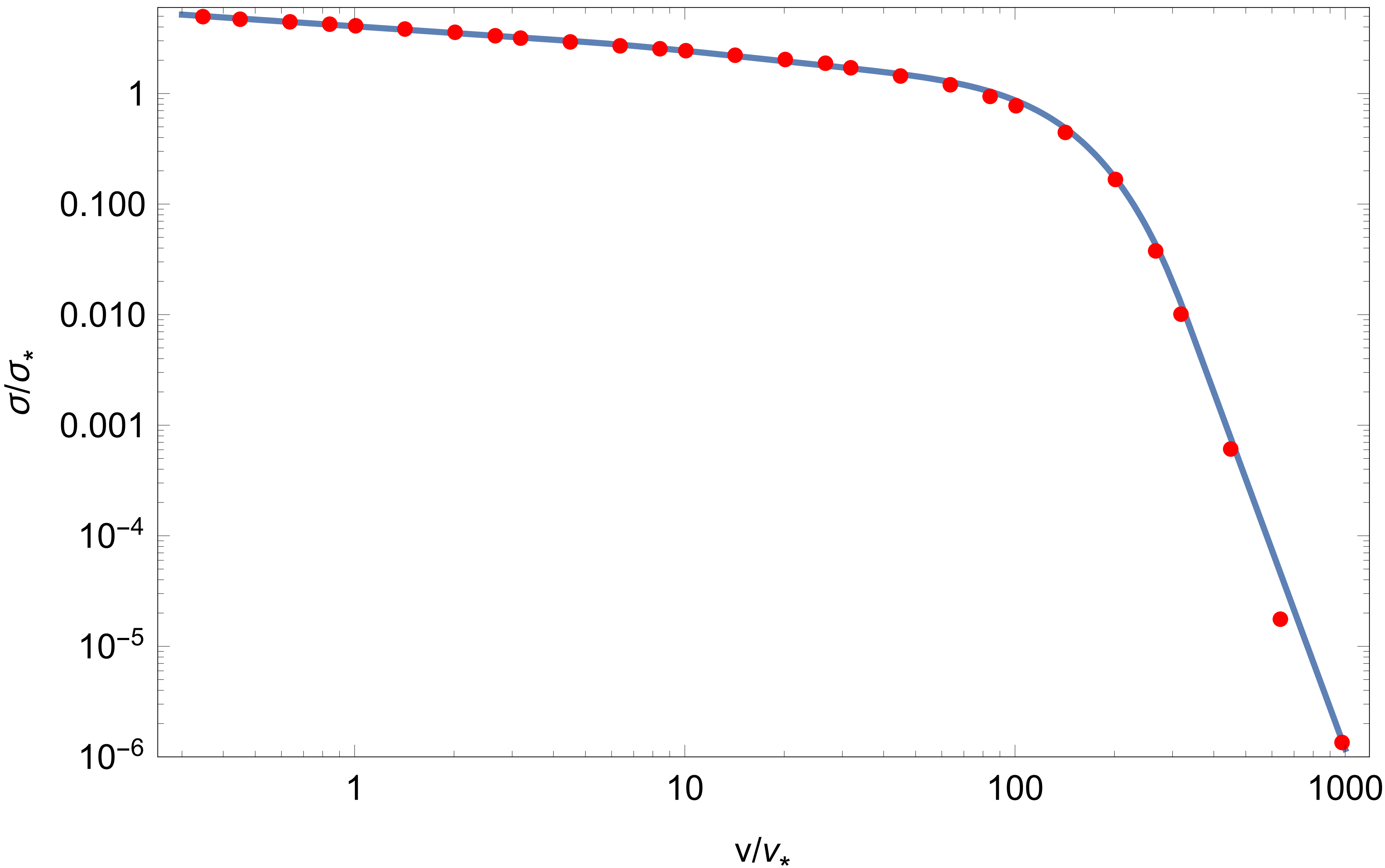
$$\begin{aligned}
\frac{\partial f_\mathrm{n}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\mathrm{n}}{\partial \mathbf{r}} + \frac{\mathbf{F}_\mathrm{n}}{m_\mathrm{n}}\cdot\frac{\partial f_\mathrm{n}}{\partial \mathbf{v}} &= X(f_\mathrm{n}, f_\mathrm{i}) + Y(f_\mathrm{i}) - Z(f_\mathrm{n}) \\
\frac{\partial f_\mathrm{i}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\mathrm{i}}{\partial \mathbf{r}} + \frac{\mathbf{F}_\mathrm{i}}{m_\mathrm{i}}\cdot\frac{\partial f_\mathrm{i}}{\partial \mathbf{v}} &= C(f_\mathrm{i}) - X(f_\mathrm{n}, f_\mathrm{i}) - Y(f_\mathrm{i}) + Z(f_\mathrm{n}) \\
\frac{\partial f_\mathrm{e}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\mathrm{e}}{\partial \mathbf{r}} + \frac{\mathbf{F}_\mathrm{e}}{m_\mathrm{e}}\cdot\frac{\partial f_\mathrm{e}}{\partial \mathbf{v}} &= C(f_\mathrm{e}) - Y(f_\mathrm{e}, f_\mathrm{i}) + Z(f_\mathrm{e}, f_\mathrm{n})
\end{aligned}$$

$$\begin{aligned}
\text{Recom: } Y(f_\mathrm{e}, f_\mathrm{i}) &= \int d\mathbf{v}'\sigma_\mathrm{r}|\mathbf{v}-\mathbf{v}'|f_\mathrm{i}(\mathbf{v}')f_\mathrm{e}(\mathbf{v}) \\
Y(f_\mathrm{i}) = Y(f_\mathrm{i}, f_\mathrm{e}) &= \int d\mathbf{v}'\sigma_\mathrm{r}|\mathbf{v}-\mathbf{v}'|f_\mathrm{e}(\mathbf{v}')f_\mathrm{i}(\mathbf{v}) \approx n_\mathrm{e}\left\langle\sigma_\mathrm{r} v_\mathrm{e}\right\rangle f_\mathrm{i}(\mathbf{v}) \\
\text{Ioniz: } Z(f_\mathrm{e}, f_\mathrm{n}) &= \int d\mathbf{v}'\sigma_\mathrm{z}|\mathbf{v}-\mathbf{v}'|f_\mathrm{e}(\mathbf{v}_\mathrm{f})f_\mathrm{i}(\mathbf{v}') - f_\mathrm{e}(\mathbf{v})f_\mathrm{n}(\mathbf{v}')] + \frac{\delta f_\mathrm{e}^{\mathrm{created}}(\mathbf{v})}{\delta t} \\
Z(f_\mathrm{n}) = Z(f_\mathrm{n}, f_\mathrm{e}) &= \int d\mathbf{v}'\sigma_\mathrm{z}|\mathbf{v}-\mathbf{v}'|f_\mathrm{e}(\mathbf{v}')f_\mathrm{n}(\mathbf{v}) \approx n_\mathrm{e}\left\langle\sigma_\mathrm{z} v_\mathrm{e}\right\rangle f_\mathrm{n}(\mathbf{v})
\end{aligned}$$

$$\begin{aligned}
\text{McWhirter 1965 } \left\langle\sigma_\mathrm{r} v_\mathrm{e}\right\rangle &= 7.57\times10^{-18}\sqrt{\frac{13.6\mathrm{eV}}{T_\mathrm{e}}}\mathrm{m}^3/\mathrm{s} \text{ (not } 0.7\times10^{-19}) \\
\text{Voronov 1997 } \left\langle\sigma_\mathrm{z} v_\mathrm{e}\right\rangle &= 2.91\times10^{-14}\frac{(13.6\mathrm{eV}/T_\mathrm{e})^{0.39}}{0.232+13.6\mathrm{eV}/T_\mathrm{e}}\exp\left(-\frac{13.6\mathrm{eV}}{T_\mathrm{e}}\right)\mathrm{m}^3/\mathrm{s}
\end{aligned}$$

## Cross section for the charge exchange operator

Atomic data for fusion vol. 1, C. F. Barnett, ORNL 1990




$v_\star = 1.38 \times 10^4 \mathrm{m/s}$ and $\sigma_\star = 10^{-19} \mathrm{m}^2$

# Moments of the charge exchange operator $(a,b)=(\mathrm{n},\mathrm{i})$ or $(\mathrm{i},\mathrm{n})$

$$X(f_a,f_b) \quad = \quad \int d\mathbf{v}'\sigma_X|\mathbf{v}-\mathbf{v}'|[f_a(\mathbf{v}')f_b(\mathbf{v})-f_a(\mathbf{v})f_b(\mathbf{v}')]$$

$$\hat{A}^{lpk}_{ab}\mathsf{N}^{lk}_a \quad = \quad \int d\mathbf{v}\hat{\mathsf{P}}^{lp}_a X(f^0_a\hat{\mathsf{P}}^{lk}_a\cdot\mathsf{M}^{lk}_a, f^0_b)$$

$$\hat{B}^{lpk}_{ab}\mathsf{N}^{lk}_b \quad = \quad \int d\mathbf{v}\hat{\mathsf{P}}^{lp}_a X(f^0_a, f^0_b\hat{\mathsf{P}}^{lk}_b\cdot\mathsf{M}^{lk}_b)$$

$$\int d\mathbf{v}'\sigma_X|\mathbf{v}-\mathbf{v}'|f^{lk}_b(\mathbf{v}') = n_b\sigma_\star\mathsf{P}^l(\hat{\mathbf{v}})\cdot\mathsf{M}^{lk}_b S^{lk}_{ab},\ \ \hat{w}_a=\frac{\frac{1}{2}m_a v^2}{T_a}=s_a^2$$

$$S^{lk}_{ab}=\frac{1}{\pi^{1/2}}\underbrace{\int_0^\infty d\hat{w}'\hat{w}'^{(l+1)/2}L^l_k(\hat{w}')e^{-\hat{w}'}}_{\mathrm{Gauss-Laguerre}}\underbrace{\int_{-1}^1 d\xi' P_l(\xi')\hat{\sigma}_X|\mathbf{v}-\mathbf{v}'|}_{\mathrm{Gauss-Legendre}}$$

$$\begin{aligned}\sigma_l A^{lpk}_{ab}\mathsf{M}^{lk}_a &= -\int d\mathbf{v}\mathsf{P}^{lp}_a(\mathbf{s}_a)\int d\mathbf{v}'\sigma_X|\mathbf{v}-\mathbf{v}'|[f^{lk}_a(\mathbf{v})f^0_b(\mathbf{v}')-f^{lk}_a(\mathbf{v}')f^0_b(\mathbf{v})]\\ &= -\sigma_l n_b\sigma_\star\mathsf{N}^{lk}_a\frac{2}{\sqrt{\pi}}\int d\hat{w}\hat{w}^{(1+l)/2}L^l_p(\hat{w})e^{-\hat{w}}\\ &\quad\times[\hat{w}^{l/2}L^l_k(\hat{w})S^{00}_{ab}-S^{lk}_{aa}(T^a_b)^{3/2}e^{\hat{w}(1-T^a_b)}]\end{aligned}$$

$$\begin{aligned}\sigma_l B^{lpk}_{ab}\mathsf{M}^{lk}_a &= -\int d\mathbf{v}\mathsf{P}^{lp}_a(\mathbf{s}_a)\int d\mathbf{v}'\sigma_X|\mathbf{v}-\mathbf{v}'|[f^0_a(\mathbf{v})f^{lk}_b(\mathbf{v}')-f^0_a(\mathbf{v}')f^{lk}_b(\mathbf{v})]\\ &= -\sigma_l n_a\sigma_\star\mathsf{N}^{lk}_b\frac{2}{\sqrt{\pi}}\int d\hat{w}\hat{w}^{(1+l)/2}L^l_p(\hat{w})e^{-\hat{w}}\end{aligned}$$

# Ion closures: kinetic equation

- Sum of electron and ion equations

$$\frac{\partial f_{\rm i}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_{\rm i}}{\partial \mathbf{r}} + \mathbf{a}_{\rm i}\cdot\frac{\partial f_{\rm i}}{\partial \mathbf{v}} + \frac{\partial f_{\rm n}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_{\rm n}}{\partial \mathbf{r}} + \mathbf{a}_{\rm n}\cdot\frac{\partial f_{\rm n}}{\partial \mathbf{v}} = C(f_{\rm i})$$

- Set $f = f^{\rm M} + f^{\rm N}$

  Solve the kinetic equation to express $f^{\rm N}$ in terms of $f^{\rm M}$

  Take closure moments to express closures in terms of thermodynamicdrives

- Asymptotic closure scheme

  Adopt the closure ordering: $\dfrac{\partial f^{\rm N}}{\partial t} = 0$

  Ignore $\mathbf{v}\cdot\dfrac{\partial f^{\rm N}}{\partial \mathbf{r}}$ and $\dfrac{q}{m}\mathbf{E}\cdot\dfrac{\partial f^{\rm N}}{\partial \mathbf{v}}$ $\Leftarrow$ only for high collisionality

$$\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}(f_{\rm i}^{\rm M} + f_{\rm n}^{\rm M}) - \frac{q_{\rm i}}{m_{\rm i}}\mathbf{v}\times\mathbf{B}\cdot\frac{\partial f_{\rm i}^{\rm N}}{\partial \mathbf{v}} = C(f_{\rm i}^{\rm M}, f_{\rm e}^{\rm M}) + C_{\rm iL}(f_{\rm i}^{\rm N}) + \text{Maxwellian}$$

# Ion closures: moment equations

- Take moments of $C_{\mathrm{iL}}(f_{\mathrm{i}}^{\mathrm{N}}) - \frac{q_{\mathrm{i}}}{m_{\mathrm{i}}}\mathbf{v}\times\mathbf{B}\cdot\frac{\partial f_{\mathrm{i}}}{\partial \mathbf{v}} = -\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}(f_{\mathrm{i}}^{\mathrm{M}} + f_{\mathrm{n}}^{\mathrm{M}}) + C(f_{\mathrm{i}}^{\mathrm{M}}, f_{\mathrm{e}}^{\mathrm{M}})$

$$-\mathsf{C}_{\mathrm{i}}^{l}\mathsf{N}_{\mathrm{i}}^{l} + \Omega_{\mathrm{i}}\mathbf{b}\breve{\times}\mathsf{N}_{\mathrm{i}}^{l} = \mathsf{G}_{\mathrm{i}}^{\nabla,l} + \mathsf{G}_{\mathrm{n}}^{\nabla,l} \;\Rightarrow\; \nabla(T_{\mathrm{i}} + T_{\mathrm{n}}) \text{ and } \mathsf{W}_{\mathrm{i}} + \mathsf{W}_{\mathrm{n}}$$

$$l = 1:\; -\frac{1}{\tau_{\mathrm{ii}}}\mathsf{c}_{\mathrm{i}}\mathsf{N}_{\mathrm{i}}^{1} + \Omega_{\mathrm{i}}\mathbf{b}\times\mathsf{N}_{\mathrm{i}}^{1} = \mathsf{g}^{1}\left(\frac{n_{\mathrm{i}}v_{T\mathrm{i}}}{T_{\mathrm{i}}}\nabla T_{\mathrm{i}} + \frac{n_{\mathrm{n}}v_{T\mathrm{n}}}{T_{\mathrm{n}}}\mathsf{g}^{1}\nabla T_{\mathrm{n}}\right),\; \mathsf{g}^{1} \equiv \begin{pmatrix} \frac{\sqrt{5}}{2} \\ 0 \\ \vdots \end{pmatrix}$$

- General solution of a set of vector equations
  Define $\mathbf{g}_{\|} \equiv \mathbf{bb}\cdot\mathbf{g},\; \mathbf{g}_{\times} \equiv \mathbf{b}\times\mathbf{g},\; \mathbf{g}_{\perp} \equiv -\mathbf{b}\times(\mathbf{b}\times\mathbf{g})$

$$-\mathsf{c}\mathbf{N} + r\mathbf{b}\times\mathbf{N} = \mathbf{g} \;\Rightarrow\; \mathbf{N} = -\left[\mathsf{c}^{-1}\mathbf{g}_{\|} + \frac{1}{\mathsf{c}^{2} + r^{2}\mathsf{I}}(r\mathbf{g}_{\times} + \mathsf{c}\mathbf{g}_{\perp})\right]$$

$$\mathsf{N}_{\mathrm{i},a}^{1} = -\frac{n_{a}v_{Ta}\tau_{\mathrm{ii}}}{T_{a}}[\mathtt{igl}\nabla_{\|}T_{a} + \mathtt{igx}\nabla_{\times}T_{a} + \mathtt{igp}\nabla_{\perp}T_{a})]$$

$$\mathsf{N}_{\mathrm{i}}^{1} = \begin{pmatrix} \mathsf{N}_{\mathrm{i}}^{11}(v^{2}\mathbf{v}) \\ \mathsf{N}_{\mathrm{i}}^{12}(v^{4}\mathbf{v}) \\ \vdots \end{pmatrix} = \mathsf{N}_{\mathrm{i,i}}^{1} + \mathsf{N}_{\mathrm{i,n}}^{1} \;\Rightarrow\; \begin{aligned} \mathbf{h} &= -\frac{\sqrt{5}}{2}Tv_{T}\mathsf{N}^{11} \\ \mathbf{R}_{\mathrm{ni}} &\propto \sum_{k=1}(\hat{A}_{\mathrm{ni}}^{10k}\mathsf{N}_{\mathrm{n}}^{1k} + \hat{B}_{\mathrm{ni}}^{10k}\mathsf{N}_{\mathrm{i}}^{1k}) \end{aligned}$$

# Neutral closures

$$\mathbf{v}\cdot\frac{\partial f_{\mathrm{n}}^{\mathrm{M}}}{\partial \mathbf{r}} = X(f_{\mathrm{n}}^{\mathrm{M}}, f_{\mathrm{i}}^{\mathrm{M}}) + X(f_{\mathrm{n}}^{\mathrm{N}}, f_{\mathrm{i}}^{\mathrm{M}}) + X(f_{\mathrm{n}}^{\mathrm{M}}, f_{\mathrm{i}}^{\mathrm{N}}) + Y(f_{\mathrm{i}}^{\mathrm{N}}) - Z(f_{\mathrm{n}}^{\mathrm{N}})$$

Take moments: $-\mathsf{G}^{l}_{\nabla,\mathrm{n}} = \mathsf{G}^{l}_{X,\mathrm{n}} + \mathsf{A}^{l}_{\mathrm{ni}}\mathsf{N}^{l}_{\mathrm{n}} + \mathsf{B}^{l}_{\mathrm{ni}}\mathsf{N}^{l}_{\mathrm{i}} + \mathsf{Y}^{l}_{\mathrm{ni}}\mathsf{N}^{l}_{\mathrm{i}} - \mathsf{Z}^{l}_{\mathrm{nn}}\mathsf{N}^{l}_{\mathrm{n}}$

$$\underbrace{\left(-\mathsf{a}^{1}_{\mathrm{ni}}+\frac{\tau_X}{\tau_Z}\mathsf{z}^{1}_{\mathrm{nn}}\right)}_{\texttt{AZ}}\mathsf{N}^{1}_{\mathrm{n}} = \frac{n_{\mathrm{n}}v_{T\mathrm{n}}\tau_X}{T_{\mathrm{n}}}\mathsf{g}^{1}\nabla T_{\mathrm{n}} + \mathsf{a}^{1.0}_{\mathrm{ni}}\frac{\sqrt{2}n_{\mathrm{n}}\mathbf{V}_{\mathrm{n}}}{v_{T\mathrm{n}}} + \mathsf{b}^{1.0}_{\mathrm{ni}}\frac{\sqrt{2}n_{\mathrm{n}}\mathbf{V}_{\mathrm{i}}}{v_{T\mathrm{i}}} + \left(\mathsf{b}^{1}_{\mathrm{ni}}\frac{n_{\mathrm{n}}}{n_{\mathrm{i}}} + \frac{\tau_X}{\tau_Y}\mathsf{y}^{1}_{\mathrm{ni}}\right)\mathsf{N}^{1}_{\mathrm{i}}$$

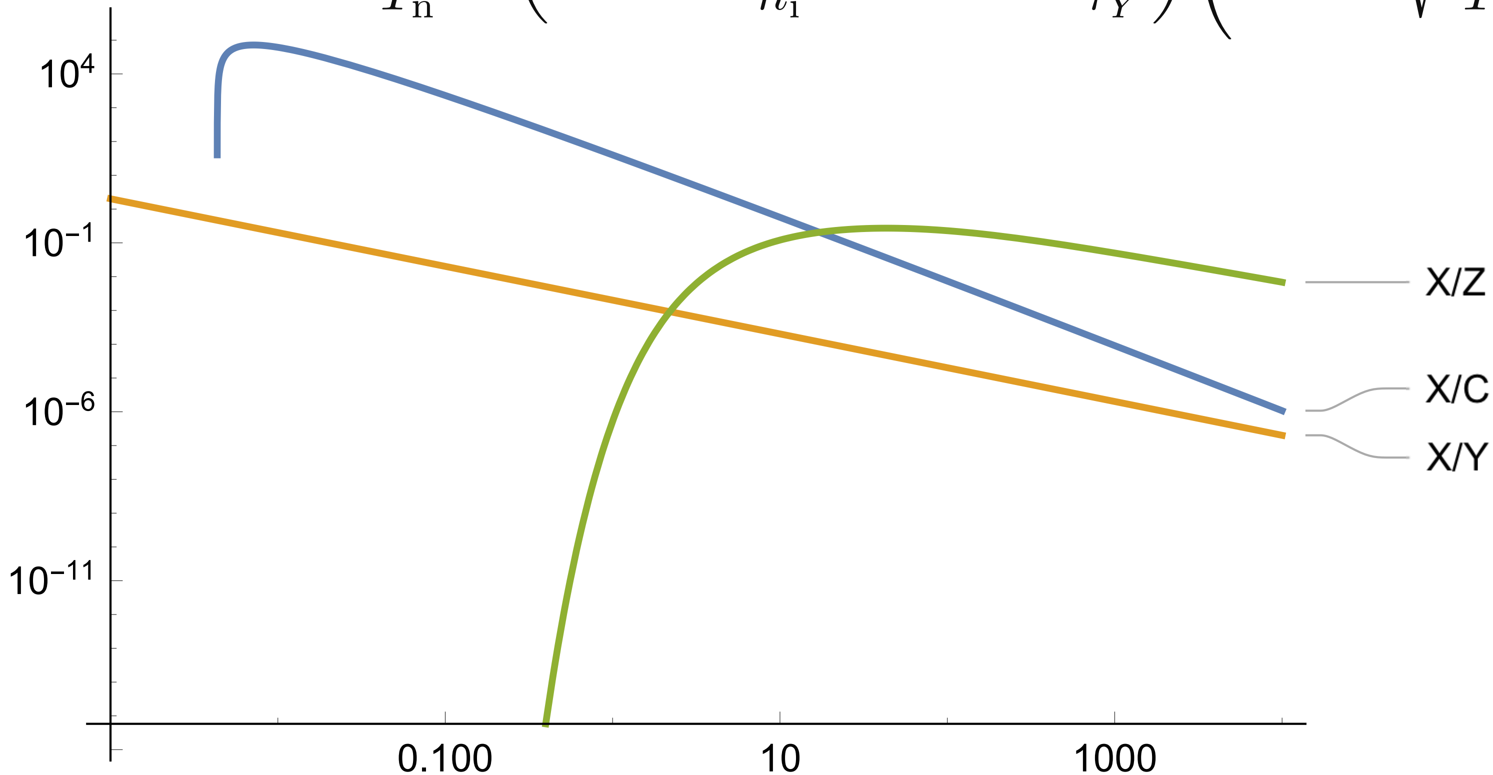
$$\begin{aligned}\mathsf{N}^{1}_{\mathrm{n}} &= \frac{n_{\mathrm{n}}v_{T\mathrm{n}}\tau_X}{T_{\mathrm{n}}}\left(\texttt{ngn}\nabla T_{\mathrm{n}} + \texttt{nvn}\frac{\sqrt{2}n_{\mathrm{n}}\mathbf{V}_{\mathrm{n}}}{v_{T\mathrm{n}}} + \texttt{nvi}\frac{\sqrt{2}n_{\mathrm{n}}\mathbf{V}_{\mathrm{i}}}{v_{T\mathrm{i}}}\right)\\ &\quad - \frac{n_{\mathrm{n}}v_{T\mathrm{n}}\tau_{\mathrm{ii}}}{T_{\mathrm{n}}}\left(\texttt{ngl}\nabla_{\parallel}T_{\mathrm{n}} + \texttt{ngx}\nabla_{\times}T_{\mathrm{n}} + \texttt{ngp}\nabla_{\perp}T_{\mathrm{n}}\right)\\ &\quad - \frac{n_{\mathrm{i}}v_{T\mathrm{i}}\tau_{\mathrm{ii}}}{T_{\mathrm{i}}}\left(\texttt{ngl}\nabla_{\parallel}T_{\mathrm{i}} + \texttt{ngx}\nabla_{\times}T_{\mathrm{i}} + \texttt{ngp}\nabla_{\perp}T_{\mathrm{i}}\right)\end{aligned}$$

$$\mathsf{N}^{1}_{\mathrm{n}} = \frac{n_{\mathrm{n}}v_{T\mathrm{n}}\tau_X}{T_{\mathrm{n}}}\left(\texttt{ngn}\nabla_{\parallel}T_{\mathrm{n}} + \texttt{nvn}\frac{\sqrt{2}n_{\mathrm{n}}\mathbf{V}_{\mathrm{ni}}}{v_{T\mathrm{n}}}\right) - \frac{n_{\mathrm{n}}v_{T\mathrm{n}}\tau_{\mathrm{ii}}}{T_{\mathrm{n}}}\texttt{ngl}\left(\nabla_{\parallel}T_{\mathrm{n}} + \sqrt{\frac{T_{\mathrm{n}}}{T_{\mathrm{i}}}}\frac{n_{\mathrm{i}}}{n_{\mathrm{n}}}\nabla_{\parallel}T_{\mathrm{i}}\right) + \ldots$$

where $\texttt{ngl} = \texttt{AZ}^{-1}\mathsf{b}^{1}_{\mathrm{ni}}\texttt{igl}\frac{n_{\mathrm{n}}}{n_{\mathrm{i}}} + \texttt{AZ}^{-1}\mathsf{y}^{1}_{\mathrm{ni}}\texttt{igl}\frac{\tau_X}{\tau_Y} \equiv \texttt{nglbigl}\frac{n_{\mathrm{n}}}{n_{\mathrm{i}}} + \texttt{nglyigl}\frac{\tau_X}{\tau_Y}$

# Neutral closures (cont.)

$$\mathsf{N}_{\rm n}^{1} = \frac{n_{\rm n} v_{T\rm n} \tau_X}{T_{\rm n}} \left( \mathtt{ngn} \nabla_\parallel T_{\rm n} + \mathtt{nvn} \frac{\sqrt{2} n_{\rm n} \mathbf{V}_{\rm ni}}{v_{T\rm n}} \right) - \frac{n_{\rm n} v_{T\rm n} \tau_{\rm ii}}{T_{\rm n}} \left( \mathtt{nglbig1} \frac{n_{\rm n}}{n_{\rm i}} + \mathtt{nglyig1} \frac{\tau_X}{\tau_Y} \right) \left( \nabla_\parallel T_{\rm n} + \sqrt{\frac{T_{\rm n}}{T_{\rm i}}} \frac{n_{\rm i}}{n_{\rm n}} \nabla_\parallel T_{\rm i} \right) + \dots$$




| $T$ (eV) | 0.03 | 1 | 10 | 100 | 1 k | 10 k |
|---|---|---|---|---|---|---|
| $\tau_X/\tau_Y$ | 0.067 | 2.0E-3 | 2.0E-4 | 2.0E-5 | 2.0E-6 | 2.0E-7 |
| $\tau_X/\tau_Z$ | 0 | 5.2E-7 | 0.12 | 0.23 | 0.050 | 0.0069 |
| $\tau_X/\tau_{\rm ii}$ | 1.6E4 | 40 | 0.57 | 0.0074 | 9.1E-5 | 1.1E-6 |

# Convergence with Gaussian nodes and moments $(T = 100\mathrm{eV})$
# Convergent results: heat flow vs. temperature

$$\mathrm{N}_{\mathrm{n}}^{1} = \frac{n_{\mathrm{n}} v_{T\mathrm{n}} \tau_X}{T_{\mathrm{n}}} \left( \mathtt{ngn} \nabla_{\parallel} T_{\mathrm{n}} + \mathtt{nvn} \frac{\sqrt{2} n_{\mathrm{n}} \mathbf{V}_{\mathrm{ni}}}{v_{T\mathrm{n}}} \right) - \frac{n_{\mathrm{n}} v_{T\mathrm{n}} \tau_{\mathrm{ii}}}{T_{\mathrm{n}}} \left( \mathtt{nglbigl} \frac{n_{\mathrm{n}}}{n_{\mathrm{i}}} + \mathtt{nglyigl} \frac{\tau_X}{\tau_Y} \right) \cdots$$

Gauss-Legendre nodes $= 10$ (or $20$), ($n_w =$Gauss-Laguerre nodes), and moments M

| ngn $(n_w = 10)$ | (20) | (40) | (20) 3M | (20) 4M | nvn (10) | (20) | (40) | 3M | 4M |
|---|---|---|---|---|---|---|---|---|---|
| 1.732 | 1.734 | 1.735 | 1.735 | 1.736 | 0.0689 | 0.0685 | 0.0684 | 0.0687 | 0.0687 |
| 0.174 | 0.173 | 0.172 | 0.179 | 0.180 | 0.0154 | 0.0150 | 0.0150 | 0.0156 | 0.0157 |
| | | | 0.049 | 0.052 | | | | 0.0049 | 0.0049 |
| | | | | 0.018 | | | | | 0.0018 |
| **nglbigl** | | | | | **nglyigl** | | | | |
| −1.371 | −1.370 | −1.369 | −1.374 | −1.375 | −3.133 | −3.137 | −3.139 | −3.140 | −3.141 |
| 0.508 | 0.508 | 0.507 | 0.543 | 0.542 | 0.633 | 0.634 | 0.635 | 0.697 | 0.696 |
| | | | −0.060 | −0.043 | | | | −0.078 | −0.054 |
| | | | | −0.021 | | | | | −0.046 |

Heat flow results (3 vector moment approximation)

| $T$ (eV) | 0.03 | 1 | 10 | 100 | 1 k | 10 k |
|---|---|---|---|---|---|---|
| ngn | 1.05 | 1.50 | 1.59 | 1.73 | 3.81 | 34.7 |
| nvn | 0.127 | 0.127 | 0.0968 | 0.0685 | 0.0391 | -0.305 |
| nglbigl | -2.09 | -2.09 | -1.75 | -1.37 | -1.74 | -1.57 |
| nglyigl | -1.85 | -2.63 | -2.84 | -3.14 | -7.01 | -76.8 |

# Future work: collisional closures for 10 or 13 fluid moment model

- 5 (1+3+1) moments: $n(1)$, $\mathbf{V}(3)$, and $T$ (or $P = p + \frac{1}{3}mnV^2$) (1)

  Closures: $\mathbf{h}$, $\boldsymbol{\pi}$, $\mathbf{R}$, $Q$ (or $\mathbf{H}$, $\mathbf{\Pi}$, $\mathbf{R} = \mathbf{C}_\mathbf{V}$, $C_{\frac{1}{2}mv^2}$)

- 10 moments: plus $\mathbf{\Pi}(5)$

$$\partial_t \mathbf{\Pi} - 2n\mathbf{FV} + \frac{4}{5}\nabla\mathbf{H} + \nabla \cdot \mathbf{\Sigma} + \Omega(\mathbf{b} \times \mathbf{\Pi} - \mathbf{\Pi} \times \mathbf{b}) = \mathsf{C}_\mathbf{\Pi}$$

  - ◇ May capture integral (nonlocal) effects for the heat flow with collisional closures

- 13 moments: plus $\mathbf{H}(3)$

$$\partial_t \mathbf{H} + \frac{1}{2}\nabla \cdot \mathsf{U}^{21} + \frac{1}{6}\nabla \mathsf{U}^{02} - \frac{5}{2}P\frac{\mathbf{F}}{m} - \frac{\mathbf{F}}{m} \cdot \mathbf{\Pi} - \frac{q}{m}\mathbf{H} \times \mathbf{B} = \mathsf{C}_\mathbf{H}$$

  - ◇ May capture integral (nonlocal) effects for the heat flow with collisional closures

# Future work

- Parallel closures
  - ◇ Making simplified formulas in $k$ space (electrons with $Z \geq 10$ Hankyu Lee)
  - ◇ Generalize to an inhomogeneous magnetic field
- Parallel closures for nonlinear and non-adiabatic phenomena
  - ◇ Implement parallel moment equations (nonlinear terms) in NIMROD
- Closures for e-i-n plasmas
  - ◇ Complete closures for ions and neutrals
  - ◇ (Long term) Compute the cross section for the electron impact ionization (atomic physics)
  - ◇ (Long term) Compute electron closures with neutrals